Error Propagation

Adapted from the Lab Manual for Physics 128 & 241

Addition and Subtraction

Suppose that you make measurements $x \pm \delta x$, $y \pm \delta y$, $z \pm \delta z$, and are given the relation

$$A = k_1 x + k_2 y + k_3 z \ , \tag{9}$$

where the k's are all constants (positive or negative). Then

$$\delta A = \sqrt{(k_1 \delta x)^2 + (k_2 \delta y)^2 + (k_3 \delta z)^2} \ , \tag{10}$$

and you report the result $A \pm \delta A$.

Multiplication and division

Suppose now that the relation is of the form

$$A = kxyz \text{ or } A = kxy/z \text{ or } A = k/xyz \ . \tag{11}$$

Then,

$$\frac{\delta A}{A} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2} \tag{12}$$

Power functions

Suppose now that the relation is of the form

$$A = kx^n \ . \tag{13}$$

Then

$$\frac{\delta A}{A} = |n| \frac{\delta x}{x} \ . \tag{14}$$