Introduction to Measurements and Error Analysis

Adapted from `www.physics.unc.edu/about/labs/content/uncertainty.pdf`

The Uncertainty of Measurements

Some numerical statements are exact: Mary has 3 brothers, and $2 + 2 = 4$. However, all measurements have some degree of uncertainty that may come from a variety of sources. The process of evaluating this uncertainty associated with a measurement result is often called uncertainty analysis or error analysis.

Properly reporting an experimental result along with its uncertainty allows other people to make judgments about the quality of the experiment, and it facilitates meaningful comparisons with other similar values or a theoretical prediction. Without an uncertainty estimate, it is impossible to answer the basic scientific question: Does my result agree with a theoretical prediction or results from other experiments? This question is fundamental for deciding if a scientific hypothesis is confirmed or refuted.

When we make a measurement, we generally assume that some exact or true value exists based on how we define what is being measured. While we may never know this true value exactly, we attempt to find this ideal quantity to the best of our ability with the time and resources available. As we make measurements by different methods, or even when making multiple measurements using the same method, we may obtain slightly different results. So how do we report our findings for our best estimate of this elusive true value? The most common way to show the range of values that we believe includes the true value is:

$$\textbf{measurement} = \textbf{best estimate} \pm \textbf{uncertainty (units)}$$

Lets take an example. Suppose you want to find the mass of a gold ring. By simply examining the ring in your hand, you estimate the mass to be between 10 and 20 grams, but this is not a very precise estimate. After some searching, you find an electronic balance that gives a mass reading of 17.43 grams. While this measurement is much more precise than the original estimate, how do you know that it is accurate, and how confident are you that this measurement represents the true value of the rings mass? Since the digital display of the balance is limited to 2 decimal places, you could report the mass as m = 17.43g ± 0.01g. Suppose you use the same electronic balance and obtain several more readings: 17.46g, 17.42g, 17.44g, so that the average mass appears to be in the range of 17.44g ± 0.02g. By now you may feel confident that you know the mass of this ring to the nearest hundredth of a gram, but how do you know that the true value definitely lies between 17.43g and 17.45g? Since you want to be honest, you decide to use another balance that gives a reading of 17.22g. This value is clearly below the range of values, so what do you do now?

To help answer these questions, we should first define the terms accuracy and precision:

Accuracy is the closeness of agreement between a measured value and a true or accepted value. Measurement error is the amount of inaccuracy.

Precision is the degree of consistency and agreement among independent measurements of the same quantity; also the reliability or reproducibility of the result.

Caution: Unfortunately the terms error and uncertainty are often used interchangeably to describe both imprecision and inaccuracy. Whenever you encounter these terms, make sure you understand whether they refer to accuracy or precision, or both.

Notice that in order to determine the accuracy of a particular measurement, we have to know the ideal, true value, which we really never do. We retain a useful definition of accuracy by assuming that, even when we do not know the true value, we can rely on the best available accepted value with which to compare our experimental value.

For our example with the gold ring, there is no accepted value with which to compare. The only way to assess the accuracy of the measurement is to compare with a known standard, such as a standard mass that is accurate within a narrow tolerance and is traceable to a primary mass standard at the National Institute of Standards and Technology (NIST). Calibrating the balances should eliminate the discrepancy between the readings and provide a more accurate mass measurement.

When analyzing experimental data, it is important that you understand the difference between precision and accuracy. Precision indicates the quality of the measurement, without any guarantee that the measurement is correct. Accuracy, on the other hand, assumes that there is an ideal value, and tells how far your answer is from that ideal, right answer. These concepts are directly related to random and systematic measurement errors.

Types of Errors

Random errors are statistical fluctuations (in either direction) in the measured data due to the precision limitations of the measurement device. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations (see standard error).

Systematic errors are reproducible inaccuracies that are consistently in the same direction. These errors are difficult to detect and cannot be analyzed statistically. If a systematic error is identified when calibrating against a standard, applying a correction or correction factor to compensate for the effect can reduce the bias. Unlike random errors, systematic errors cannot be detected or reduced by increasing the number of observations.

Estimating Experimental Uncertainty for a Single Measurement

Any measurement you make will have some uncertainty associated with it, no matter how precise your measuring tool. How do you actually determine the uncertainty, and once you know it, how do you report it?

For example, if you are trying to use a meter stick to measure the diameter of a tennis ball, the uncertainty might be ± 5 mm, but if you used a Vernier caliper, the uncertainty could be reduced to maybe ± 2 mm. The limiting factor with the meter stick is parallax, while the second case is limited by ambiguity in the definition of the tennis balls diameter (its fuzzy!). In both of these cases, the uncertainty is greater than the smallest divisions marked on the measuring tool (likely 1 mm and 0.1 mm respectively). Unfortunately, there is no general rule for determining the uncertainty in all measurements. The experimenter is the one who can best evaluate and quantify the uncertainty of a measurement based on all the possible factors that affect the result. Therefore, the person making the measurement has the obligation to make the best judgment possible and report the uncertainty in a way that clearly explains what the uncertainty represents:

$$\text{Measurement} = (\text{measured value} \pm \text{standard uncertainty}) \text{ unit of measurement}$$

where the ± standard uncertainty indicates approximately a 68% confidence interval (see sections on Standard Deviation and Reporting Uncertainties).

$$\text{Example: Diameter of tennis ball} = 6.7 \pm 0.2 \text{ cm}$$