Astronomy 160 Winter 2007: Syllabus

160.002:	Tuesday	7 PM - 9 PM	AH 5180B
160.003:	Thursday	7 PM - 9 PM	AH 5179

Contact Information

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I am frequently available to meet outside of office hours, although I recommend making an appointment first. I am also willing to answer questions via email; however, they must be specific and you must allow me 24 hours to respond.

Attendance and Grades

Attendance in lab is mandatory. Missing a lab will result in zero credit for that week. If you have a legitimate conflict with attending lab (religious conflict or illness), please let me know as far ahead of time as possible, and alternate arrangements *may* be possible.

Your lab grade (which is 25% of your grade for the course) consists of

- 7% Notebooks. You will be required to keep a notebook of the data and observations you take during lab. The notebooks will be collected three times throughout the semester. If your notebook is neat and complete, it will boost your overall grade.
- 18% Reports. You are required to **type** a brief report of each lab and turn in this report one week later. You will be graded on the content of your report as well as its clarity and brevity.

No late work will be accepted.

Plagiarism

We will frequently work in pairs during lab, but each person **must** turn in a separate, distinct report. Furthermore, you may cite, but not copy, the textbook. To further familiarize yourself with what constitutes plagiarism, please explore the following website:

`http://www.astro.lsa.umich.edu/Course/plagiarism.php`

Facilities

Planetarium Rules

The projector is expensive, fragile, and at elbow level, so there are some important rules. Ignoring these rules and causing damage to the planetarium may result in either you or I being billed for repairs (which includes the cost of flying the technician in from Germany.)

- **Do not touch the projector.**
- **No food or drink in the planetarium at any time by anyone.**
- **Stay seated until the lights are on and the projector is upright and stationary.**
- **Do not attempt to enter the planetarium while the "In Use" sign is on. You will not be admitted.**

Observatory Rules

The observatory poses some safety risks (i.e., falling off the roof), so please exercise caution and observe the following rules:

- **No smoking, food, or drink in the dome or on the roof.**
- **Do not move the 0.4-m (big) telescope unless asked, especially if someone is looking in the eyepiece.**

Lab Work

Lab Activities

Each week I will give you a handout for the next week's lab; you should read the lab over and make sure you understand roughly what we will be doing. **I reserve the right to administer pop quizzes if I find students are not doing this reading.** These pop quizzes would then factor into your notebook grade.

It is important for you to bring **a pen, a scientific calculator, and your notebook** to every lab. You may also find the textbook helpful at times.

Notebooks

The lab notebook is a permanent record of your data. The ability to keep a good notebook is invaluable in many experimental fields of science. **It should be legible, written in pen, with numbered pages, and initialed by me before you leave.**

Your notebook should contain the following:

- **Table of Contents:** Use the first page of your notebook for a table of contents, and keep it updated through the semester.
- **Handouts:** Permanently affix each handout I give you into your notebook, including this syllabus.
- **Lab Header:** Each week, record the date and your lab partner's name.
- **Data:** Consists of observations as well as measurements.

Reports

Each week you will turn in a **typed** report on the lab we did the previous week. This should not be a long report. **Reports should be no longer than 2 pages, assuming 12 pt font and 1 inch margins.** Your report should consist of:

- **Header:** Your name, the lab, etc.
- **Objectives:** This should be a **short** (roughly two sentences) section saying what the point of the lab is. If you see no point to the lab, lie. Be aware that learning to use a piece of equipment is never the objective of an experiment or observation.
- **The Rest:** Here, type up your data (in neat tables), your graphs and calculations (using an equation editor), and analysis (generally, questions the handout asks of you). Be aware that no questions or directives in the handout are rhetorical. Use complete sentences, even when answering a yes or no question.
- **End Comment:** State how many hours you spent on this report as well as any comments you may have on the lab.

The Celestial Sphere

Adapted from `http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php`

Introduction

Here on Earth we have a coordinate system called latitude and longitude that allow us to pinpoint any location on the globe. Of course, this isn't always useful: giving the latitude and longitude of the grocery store won't help most people find it. Hence we also use local coordinates with instructions like "2.3 miles west of my apartment."

The **equatorial system** is analogous to the latitude and longitude system we use on Earth. To map out these lines, astronomers start by projecting the Earth's north and south poles and the equator onto the sky. Those projections are called the north and south celestial poles and the celestial equator. Lines of Right Ascention (RA) run through the north and south celestial poles, crossing the equator at right angles just like lines of longitude. Lines of declination circle the sky perpendicular to this like latitude lines. In fact, the declination lines match the latitude lines: the celestial equator is at 0° dec, the north celestial pole at +90° or 90° north dec, and the south celestial pole at -90° or 90° south dec. Since Ann Arbor is at a latiude of +42.3° the declination of zenith is +42.3°. Note the fractional degree may be given as a decimal, or in minutes and seconds of arc. There are 60 arcminutes in a degree, 60 arcseconds in an arcminute.

The right ascension is a little harder. On Earth, 0° longitude is measured from Greenwich, England. We set our watches by the motion of the sun (solar time) again using Greenwich as the standard (universal time) so the Sun should transit the meridian at roughly noon every day in Greenwich. We choose the vernal equinox to be our celestial "Greenwich," where RA is equal to zero. We measure RA in hours; that way, your local sidereal time is equal to the RA of an object transiting the meridian. For example, if your local sidereal time is 20:15, a star at 20h 15m 0s RA would be on the meridian. This also makes it convenient to figure out how long an object will remain visible. Coordinates are normally given with RA first, then dec: the summer solstice is located at 6h 0m 0s and 23° 30m.

Just as the latitude and longitude aren't always convenient on Earth, the equatorial system isn't always convenient for astronomers. So there is an alternate: the altitude and azimuth system. The altitude is ideally measured in degrees above the horizon. An object at zenith is at 90°, and object on the horizon is at 0°. (Be careful if you are on a mountaintop or in a valley!) In the field, altitude is usually estimated using your hands for reference. The Moon subtends an angle of 0.5°, which is about the width of your index finger held at arm's length. Three fingers take up about 5°, your fist is about 10°, and if you spread your hand out, from index finger to pinky is about 15° and from thumb to pinky is about 20°. However, in the planetarium we have projected scales we will use.

The azimuth is measured in degrees away from north. The system is the same as the markings on a compass: 90° is due East, 180° is due South, and 270° due West. Coordinates are usually give with altitude first, azimuth second: the position of the Sun on the summer solstice when it transits the meridian is 65.8° at 180°

As you work on this activity try to keep in mind that one of these coordinate systems is "global", completely independant of your location, while the other is "local", totally dependant on your location.

Important Elements of the Celestial Sphere

Your GSI will point out components of the celestial sphere. As she does so, note the following elements: the zenith, the horizon, the celestial north pole, the celestial equator, the ecliptic, and the equinoxes. In your notebook, record some observations regarding these elements. More detailed observations will better help you to answer questions later.

Alt-Az: Altitude and Azimuth

Make a table as follows:

Star Name	Position	Altitude (deg)	Azimuth (deg)

Your GSI will set the planetarium to display the stars tonight at 8 PM and point out a star to watch north of the equator. Record the star's name as well as the altitude and azimuth for position 1 in your table.

Your GSI will move the projector until the star transits the meridian. Record the altitude and azimuth under position 2 for the first star.

Your GSI will again move the planetarium back to 8 PM and point out a star south of the equator to watch. Record its name and the altitude and azimuth for position 1 in your table. Your GSI will move the planetarium again until the second star transits. Record the altitude and azimuth for position 2 for this second star.

Finally, think about how accurate your measurements are. Could you numbers be 1 degree off? 2 degrees? 10? Determine the maximum amount by which your measurement of the position could vary, and record this as your uncertainty.

1. Does the altitude of most of the stars remain constant or change throughout the night? Does the azimuth? Make sure to take the uncertainty in your measurements into account when answering this question.
2. What object(s) has/have (roughly) the same alt-az all the time? What are its coordinates or their range of coordinates?
3. What is the possible range of altitude in degrees in the planetarium (look carefully at the meridian markings!)?
4. What is the possible range of azimuth in degrees in the planetarium?

Equatorial: Right Ascension and Declination

Now you will observe the same 2 stars you did in part 1, but this time measure their Right Ascension and declination. Record your results in a second table:

Star Name	Position	RA (hours)	Dec (deg)

Once again, determine the uncertainty in your measurements.

5. Does the RA of most of the stars remain constant or change throughout the night? Does the dec? Make sure to take the uncertainty in your measurements into account when answering this question.
6. What object(s) has/have (roughly) the same RA and dec all the time?
7. What is the possible range of RA in hours visible from Ann Arbor? Why do we use hours instead of degrees?
8. What is the possible range of dec visible in Ann Arbor?

Thinking Problems

9. What is the altitude and azimuth of Polaris (as viewed from Ann Arbor)? What is its RA and dec?
10. On what date is the longest day in Ann Arbor? In Sydney, Australia? Near Quito, Ecuador?

Introduction to Measurements and Error Analysis

Adapted from `www.physics.unc.edu/about/labs/content/uncertainty.pdf`

The Uncertainty of Measurements

Some numerical statements are exact: Mary has 3 brothers, and 2 + 2 = 4. However, all measurements have some degree of uncertainty that may come from a variety of sources. The process of evaluating this uncertainty associated with a measurement result is often called uncertainty analysis or error analysis.

Properly reporting an experimental result along with its uncertainty allows other people to make judgments about the quality of the experiment, and it facilitates meaningful comparisons with other similar values or a theoretical prediction. Without an uncertainty estimate, it is impossible to answer the basic scientific question: Does my result agree with a theoretical prediction or results from other experiments? This question is fundamental for deciding if a scientific hypothesis is confirmed or refuted.

When we make a measurement, we generally assume that some exact or true value exists based on how we define what is being measured. While we may never know this true value exactly, we attempt to find this ideal quantity to the best of our ability with the time and resources available. As we make measurements by different methods, or even when making multiple measurements using the same method, we may obtain slightly different results. So how do we report our findings for our best estimate of this elusive true value? The most common way to show the range of values that we believe includes the true value is:

measurement = best estimate ± uncertainty (units)

Lets take an example. Suppose you want to find the mass of a gold ring. By simply examining the ring in your hand, you estimate the mass to be between 10 and 20 grams, but this is not a very precise estimate. After some searching, you find an electronic balance that gives a mass reading of 17.43 grams. While this measurement is much more precise than the original estimate, how do you know that it is accurate, and how confident are you that this measurement represents the true value of the rings mass? Since the digital display of the balance is limited to 2 decimal places, you could report the mass as m = 17.43g ± 0.01g. Suppose you use the same electronic balance and obtain several more readings: 17.46g, 17.42g, 17.44g, so that the average mass appears to be in the range of 17.44g ± 0.02g. By now you may feel confident that you know the mass of this ring to the nearest hundredth of a gram, but how do you know that the true value definitely lies between 17.43g and 17.45g? Since you want to be honest, you decide to use another balance that gives a reading of 17.22g. This value is clearly below the range of values, so what do you do now?

To help answer these questions, we should first define the terms accuracy and precision:

Accuracy is the closeness of agreement between a measured value and a true or accepted value. Measurement error is the amount of inaccuracy.

Precision is the degree of consistency and agreement among independent measurements of the same quantity; also the reliability or reproducibility of the result.

Caution: Unfortunately the terms error and uncertainty are often used interchangeably to describe both imprecision and inaccuracy. Whenever you encounter these terms, make sure you understand whether they refer to accuracy or precision, or both.

Notice that in order to determine the accuracy of a particular measurement, we have to know the ideal, true value, which we really never do. We retain a useful definition of accuracy by assuming that, even when we do not know the true value, we can rely on the best available accepted value with which to compare our experimental value.

For our example with the gold ring, there is no accepted value with which to compare. The only way to assess the accuracy of the measurement is to compare with a known standard, such as a standard mass that is accurate within a narrow tolerance and is traceable to a primary mass standard at the National Institute of Standards and Technology (NIST). Calibrating the balances should eliminate the discrepancy between the readings and provide a more accurate mass measurement.

When analyzing experimental data, it is important that you understand the difference between precision and accuracy. Precision indicates the quality of the measurement, without any guarantee that the measurement is correct. Accuracy, on the other hand, assumes that there is an ideal value, and tells how far your answer is from that ideal, right answer. These concepts are directly related to random and systematic measurement errors.

Types of Errors

Random errors are statistical fluctuations (in either direction) in the measured data due to the precision limitations of the measurement device. Random errors can be evaluated through statistical analysis and can be reduced by averaging over a large number of observations (see standard error).

Systematic errors are reproducible inaccuracies that are consistently in the same direction. These errors are difficult to detect and cannot be analyzed statistically. If a systematic error is identified when calibrating against a standard, applying a correction or correction factor to compensate for the effect can reduce the bias. Unlike random errors, systematic errors cannot be detected or reduced by increasing the number of observations.

Estimating Experimental Uncertainty for a Single Measurement

Any measurement you make will have some uncertainty associated with it, no matter how precise your measuring tool. How do you actually determine the uncertainty, and once you know it, how do you report it?

For example, if you are trying to use a meter stick to measure the diameter of a tennis ball, the uncertainty might be ± 5 mm, but if you used a Vernier caliper, the uncertainty could be reduced to maybe ± 2 mm. The limiting factor with the meter stick is parallax, while the second case is limited by ambiguity in the definition of the tennis balls diameter (its fuzzy!). In both of these cases, the uncertainty is greater than the smallest divisions marked on the measuring tool (likely 1 mm and 0.1 mm respectively). Unfortunately, there is no general rule for determining the uncertainty in all measurements. The experimenter is the one who can best evaluate and quantify the uncertainty of a measurement based on all the possible factors that affect the result. Therefore, the person making the measurement has the obligation to make the best judgment possible and report the uncertainty in a way that clearly explains what the uncertainty represents:

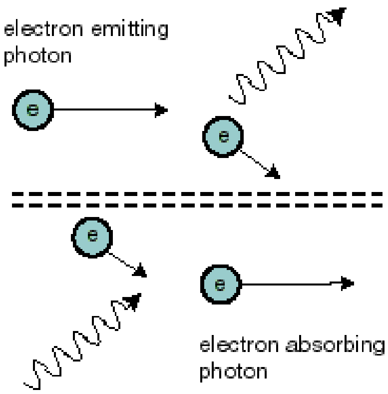
$$\text{Measurement} = (\text{measured value} \pm \text{standard uncertainty}) \text{ unit of measurement}$$

where the ± standard uncertainty indicates approximately a 68% confidence interval (see sections on Standard Deviation and Reporting Uncertainties).

Example: Diameter of tennis ball = 6.7 ± 0.2 cm

Principles of Spectroscopy

Introduction



A **photon** is a small bit of electromagnetic energy sent across space. Photons can be emitted or absorbed by electric charges – usually an **electron**.

A hot, dense object contains many "loose" electrons which can emit photons of any energy. The light produced is called **continuous emission** because it contains photons of all energies, i.e. light of all colors, or wavelengths. Hotter objects contain more energetic electrons, which in turn tend can emit more energy overall. This is described by the **Stephan-Boltzmann Law:**

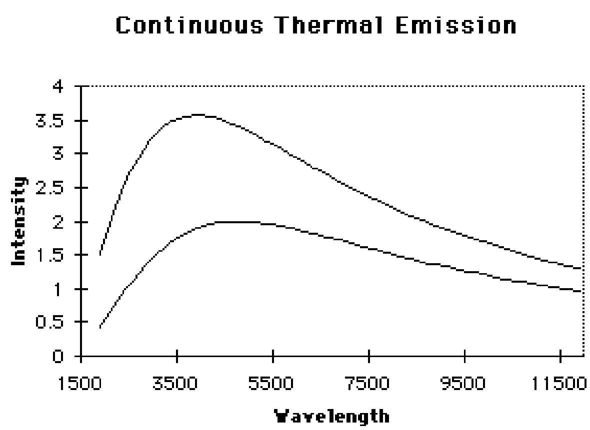
$$f = \sigma \, T^4,$$

where f is the **flux** of light energy emitted per unit area, or brightness, T is the temperature, and σ is the Stefan-Boltzmann constant, 5.67×10^{-8}J m^{-2} K^{-4}. If two objects are the same size, but one is twice as hot as the other, the hotter one will be sixteen times brighter.

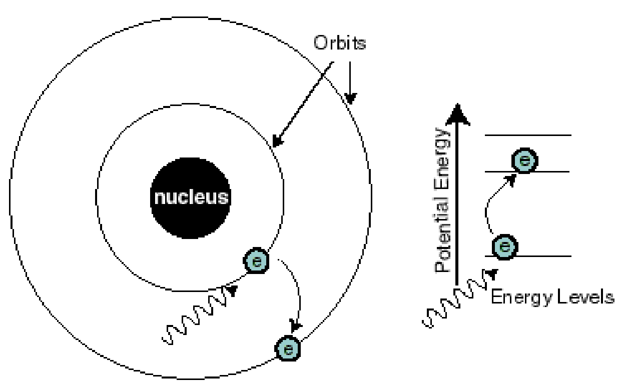
As an object heats up, the energy of the typical photon emitted increases, and the continuous emission shifts toward shorter wavelengths (higher energies) and therefore looks bluer. This is described by Wien's Law:

$$\lambda_{\text{peak}} \cdot T = 0.29 \text{ cm K}.$$

Any hot, opaque object will produce continuous emission, with the total energy and dominant color described by these two laws. This is called **blackbody radiation** or **thermal radiation**. Remember that Wien's law and the Stefan-Boltzmann Law apply only to continuous thermal emission.



We've described "loose" electrons in a hot medium. However, if electrons are part of an atom, they can only possess certain energies. As a result, these electrons emit photons that only possess certain energies, and produce emission and absorption spectra.



Continuous Emission

Get a dispersion grating from your instructor. A dispersion grating does the same thing as a prism: it splits up the light into individual wavelengths so that you can see the spectrum. Use the grating to observe the spectrum of the incandescent bulb.

1. What kind of light-source are you looking at: thin gas, opaque gas, solid, or liquid? What type of spectrum should this produce?
2. Observe the first order spectrum with the diffraction grating. What kind of spectrum is it: continuous, line emission or absorption? How did you identify it as this type of spectrum?
3. How does the brightness change with voltage?
4. How does the peak color change with voltage?
5. Explain using both Wien's and the Stephan-Boltzmann Laws: if two stars are the same size, which is *brighter*, a red star or a blue star?

Line Emission

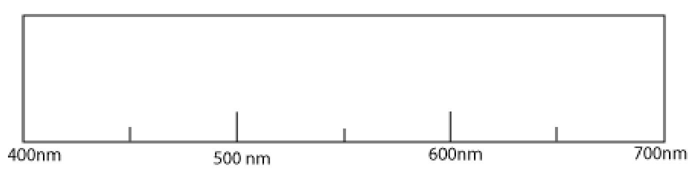
Now we will look at "discharge tubes," which are each filled with a low-density gas made of a single kind of atom. Running an electric current through the discharge tube kicks the electrons up to a high energy level. The electrons quickly fall back to their original energy level, emitting a photon with a wavelength determined by the difference in energy between the levels.

6. What kind of light-source are you looking at: thin gas, opaque gas, solid, or liquid? What type of spectrum should this produce?

There should be a spectroscope set up to observe the spectrum. A slit is aligned with the light source that allows light to travel down to the diffraction grating at the eyepiece. The spectrum is projected onto a scale to the left of the light source. Observe the spectrum through the spectroscope.

7. What kind of spectrum is it: continuous, line emission or absorption? How did you identify it as this type of spectrum?
8. Observe the spectrum of at least four of the discharge tubes, including hydrogen and the sodium lamp. Roughly sketch what you see, labeling the element's name and the colors of the brightest lines. Compare these to the chart of emission lines in the classroom. PLEASE TURN OFF THE DISCHARGE TUBES WHEN NOT IN USE (but leave the sodium lamp on)

   Example:



9. The hydrogen atom has only one electron, but when you look through the spectroscope, you see several emission lines. Explain how this is possible.
10. How are emission lines useful to an observer?

Line Absorption

It is difficult to replicate a cold thin gas with sufficient density for visible absorption lines in the lab, so we will use solid filters to observe an absorption spectrum. Since these are solids, they produce very broad absorption lines, not quite the same as the absorption patterns from Kirchoffs law.

To use the filters, hold a filter in one hand and the diffraction grating in the other. Look through the diffraction grating at the spectrum, then place the filter between the grating and your eye.

11. What kind of material is the light source: transparent gas, opaque gas, solid or liquid ? What kind of spectrum will it produce?

12. You are looking at it through air, which is a thin transparent gas. What kind of spectrum should you see? Why dont you see it?

13. Look at the light source through one of the colored filters. What kind of spectrum do you see? Explain how you determined it must be this type.

14. Observe the spectrum with at least 5 different filters, including the Hα. Make a table with the color of the filter (the plastic ones are primaries, the ones in the slide holders have their names printed on them) and what color(s) it blocked.

Filter Color	Color(s) Blocked

15. Look through the Hα filter at the discharge tubes and sodium lamp. Which light is NOT significantly blocked by the filter?

16. Why would this filter be useful to astronomers?

Thinking Questions

17. Describe two ways astronomers could determine the composition of a planets atmosphere without leaving Earth (Remember that emission and absorption lines are not restricted to the visible spectrum.)

18. Notice that the discharge tubes have different colors to our eyes. Can we use Wien's Law to tell the relative temperatures of the gas within the tubes? Explain.

Kepler's 3$^{\text{rd}}$ Law and the Mass of Jupiter

Adapted from `http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php`

Introduction

From Chapter 5, Kepler's third law for two bodies orbiting each other is

$$\left[\frac{R}{1\ \text{AU}}\right]^3 = \left[\frac{m_1 + m_2}{1\ M_\odot}\right]\left[\frac{P}{1\ \text{yr}}\right]^2 , \qquad (1)$$

where R is the distance between bodies, m_1 and m_2 are the masses of the two bodies, and P is the length of time to complete one orbit. If $m_1 \gg m_2$, what approximation can you make?

In the 1600s, Galileo used a telescope to discover that Jupiter had four moons and made exhaustive studies of this system. The Jupiter system was especially important because it is, gravitationally, a "miniature" solar system. We will use this mini solar system to test Kepler's third law.

Directions

The CLEA software simulates a telescope, with which you "observe" the moons. Open the **CLEA Labs** folder and double click on the CLEA icon: **Moons of Jupiter** and **Login**. After entering your names, choose the **Start** option to set the starting date and time. You will this table later to reset the **Interval Between Observations**.

Jupiter is in the center of the screen, with its moons to either side. The current magnification is displayed in the upper left hand corner of the screen. The date, UT (the time in Greenwich, England) and JD (Julian Date) are displayed in the lower left hand corner.

Click on each moon, using the highest magnification possible, to find the distance between the moon and Jupiter in Jupiter diameters, x. Determine the corresponding uncertainty in x. If the moon is behind Jupiter, record the distance for that moon to be zero. If this is the case, note that your uncertainty will be larger.

For each moon, make a table as follows:

Moon Name		
t (days)	x (j.d.)	δx (j.d.)

At each observation interval record how many days (may be fractional) have elapsed, the position of the moon, and the uncertainty in the position. To be consistent, give any observations east of Jupiter a negative sign and observations west of Jupiter a positive sign. Give sufficient time coverage to all four orbits. For instance, since Io's orbital period is significantly shorter than that of Callisto, you will have to change your observation interval to both get good time coverage and to make efficient use of your observing time. You need to cover a full period of the orbit from, for example, the moon's most eastern position to its most western position *and back*.

1. Using your choice of graphing program, plot each moon's position as a function of time, displaying your uncertainties as vertical error bars.

2. The data should take the shape of a sine curve. Determine the period, P, and the distance from Jupiter, R, for each moon. Estimate your uncertainty in each value of R.

3. You now have 4 sets of (P, R) coordinates, with corresponding uncertainties. Now make yet another plot, with $\ln(R/\text{AU})$ on the y-axis and $\ln(P/\text{yr})$ on the x-axis.

4. Find the slope using equation 3. Find the uncertainty in the calculated slope using equation 6. Show your work. Check your calculation using a program such as Excel. What is the expected value of the slope? Does it agree with your value? What does this test?

5. Find the intercept using equation 2. Show your work. Check your calculation using a program such as Excel. Find the uncertainty in the calculated intercept using equation 5. What is the expected value of the intercept? Does it agree with your value? What does this test?

   Note that the accepted value of M_J is 1.8986×10^{27} kg.

Thinking Questions

6. There is one glaring assumption in our computerized observing method. What is it? How could you get such even time coverage?

Least-Squares Fit to a Straight Line

Adapted from *Data Reduction and Error Analysis for the Physical Sciences*, by Philip R. Bevington and D. Keith Robinson.

Suppose that you take measurements (x_i, y_i, σ_i), where σ_i is the uncertainty in the measured value y_i. Then, the straight line that best approximates this data has slope a and intercept b given by:

$$b = \frac{1}{\Delta}\left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2}\right) \tag{2}$$

$$a = \frac{1}{\Delta}\left(\sum \frac{1}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2}\right) \tag{3}$$

$$\Delta = \sum \frac{1}{\sigma_i^2} \sum \frac{x_i^2}{\sigma_i^2} - \left(\sum \frac{x_i}{\sigma_i^2}\right)^2 \tag{4}$$

Furthermore, the uncertainty in these values is given by:

$$\sigma_b = \sqrt{\frac{1}{\Delta} \sum \frac{x_i^2}{\sigma_i^2}} \tag{5}$$

$$\sigma_a = \sqrt{\frac{1}{\Delta} \sum \frac{1}{\sigma_i^2}} \tag{6}$$

Refracting Telescopes

Adapted from `http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php`

Introduction

There are two basic types of telescopes: reflectors and refractors. The type is determined by what collects the light: reflectors have a primary mirror, while refractors have an objective lens. If the image is to be viewed directly, a lens called an eyepiece is also used. Otherwise, an astronomer may place a camera (film or CCD), spectroscope or other device on the telescope.

The focal length of a lens or mirror is given by:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{7}$$

where f is the focal length, d_o is the distance to the object (e.g. a star) and d_i is the distance to the image. Note this is for a single lens or mirror: you'll explore what happens when you add an eyepiece latter. The magnification by definition is simply the ratio of the size of the image h_i to the size of the object h_o:

$$M = \frac{h_i}{h_o} \tag{8}$$

Simple Lenses

- Note the lenses are in holders and the holders are labeled with a colored sticker with a number and letter on it. Record the color and number of your set.
- Begin by solving the equation for the focal length for f if d_o is at infinity.
- Set up the optics bench with lens B in the middle and the screen. Point the bench toward the light source. Make a table as follows:

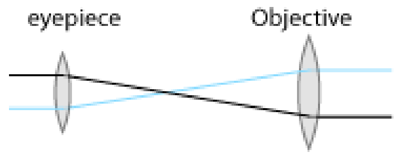
Label	d_i (cm)

- Slide the screen back and forth until the image is in focus. Record the distance between the screen and the lens as d_i in the row labeled B in your table. Repeat the same procedure for the other 4 lenses.
- Rank your lenses in order from shortest to longest focal length.
- Measure the "height" of the object. In this case, the height will be the longest dimension, even if the source is at an angle. Check with your GSI if you're unsure what to measure. Don't forget to estimate your uncertainty.
- Place your longest focal length lens on the optics bench. Measure the "height" of the image and record it as h_i. If the image is upside down, make h_i negative. Estimate your uncertainty.
- Calculate the magnification of this lens. Also, calculate your uncertainty in the magnification using error propagation methods.

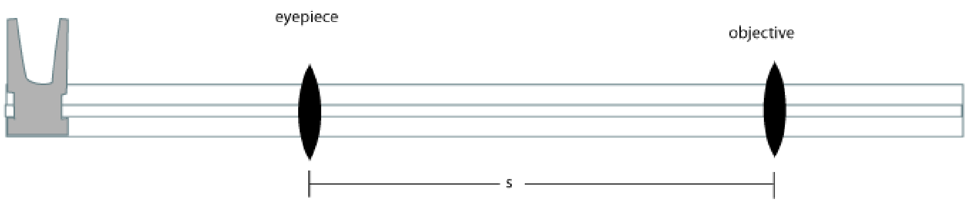
1. What does a negative magnification mean?
2. What does it mean if the absolute value of the magnification is less than 1?

Refracting Telescopes

Refractors usually have a large lens to collect the light at the front (objective), then an eyepiece to focus the light for your eye. They are designed to work for objects far enough away that the incoming light rays are parallel.



In this section you'll build a couple refractors and determine the characteristics. In order to test your telescope, you'll need to have as long a view as possible, such as down the hallway, out a window or into the next room. Your telescope should look something like this:



1. Unplug the light source from the power supply and wrap up the cord so it's out of the way. Find a comfortable place to set up where you can see a long distance and you aren't likely to loose a lens or hit one of your lab partners.
2. Choose a lens at random and slide it onto the bench. Leave just enough space between it and the light source to get your head in there to look through it: this will be the eyepiece. Record its label and focal length in your table under the eyepiece section.
3. Slide another lens onto the bench. Record its label and focal length under the objective lens.
4. For steps 5 - 7, each person will record his or her own observations. Your measurements should all be close, but small variations are expected.
5. Look through your telescope. Bring the eyepiece close to your eye, then adjust the objective to focus the image. Record the distance between the two lenses as s in your table.
6. Estimate the magnification and field of view (FofV). To estimate the magnification, keep both eyes open and try to align the two images side-by-side. For the field of view, compare it to what you could see if you didnt have the lens in the lens holder: 1 = same, 2 = twice as much, 1/2 = half as much Record your estimates in your table. If the image is upside down, record M as negative.
7. Repeat steps 2 7 using the same 2 lenses, but switch their positions (the eyepiece lens becomes the objective, the objective becomes the eyepiece for telescope number 2).
8. Repeat steps 2 - 7 with a different set of lenses.

Make a table with your measurements as follows:

eyepiece		objective		telescope properties		
Label	f (cm)	Label	f (cm)	s (cm)	M	F of V

Use the data in your table to determine the relationship between the focal lengths of the lenses and their separation and the telescope's magnification. Write those relationships as equations. Get them checked by your GSI before you continue.

Design Your Own Telescope

1. Based on your observations and relationships from Part 2, describe the properties of a telescope (s, M and FofV) with lenses of equal focal length, f :
2. Design a telescope to get the widest possble field of view. Enter the labels and focal lengths of your chosen lenses in your table.
3. Calculate the predicted separation and magnification of your telescope and enter those values in your table.
4. Test your 'scope: place the lenses on the optics bench in the correct order and look through it. Move the eyepiece to focus, and record the observed separation in your table. Estimate the magnification and record it in your table.
5. Design a second telescope to get the highest magnification possible. Repeat the steps above to fill in the bottom row of your table.

	eyepiece		objective		telescope properties		
goal	Label	f (cm)	Label	f (cm)	s (cm)	M	F of V
Widest F of V							
Highest M							

6. Look through your highest magnification telescope again and describe the view: is the image right side up or upside down, forwards or backwards, brighter or dimmer than looking directly at the source, is the entire field in focus, is there any distortion, extra or missing colors or anything else different about the source?
7. How did you decide what lenses to use and which one to make the objective and which one the eyepiece?

Thinking Questions

1. The brightness of an object generally changes the same way the field of view changes. What happens to the field of view as the magnification is increased? If you wanted to observe a faint diffuse object would you want a high or low magnification? Why?
2. These "telescopes" had identical diameter objectives and eyepieces on carriers that you could move and even change. In a real telescope, which one can you change?
3. Why is magnification not important, either to astronomers or if you were going to buy a telescope?

Error Propagation

Adapted from the Lab Manual for Physics 128 & 241

Addition and Subtraction

Suppose that you make measurements $x \pm \delta x$, $y \pm \delta y$, $z \pm \delta z$, and are given the relation

$$A = k_1 x + k_2 y + k_3 z \ , \tag{9}$$

where the k's are all constants (positive or negative). Then

$$\delta A = \sqrt{(k_1 \delta x)^2 + (k_2 \delta y)^2 + (k_3 \delta z)^2} \ , \tag{10}$$

and you report the result $A \pm \delta A$.

Multiplication and division

Suppose now that the relation is of the form

$$A = kxyz \text{ or } A = kxy/z \text{ or } A = k/xyz \ . \tag{11}$$

Then,

$$\frac{\delta A}{A} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2} \tag{12}$$

Power functions

Suppose now that the relation is of the form

$$A = kx^n \, . \tag{13}$$

Then

$$\frac{\delta A}{A} = |n| \frac{\delta x}{x} \, . \tag{14}$$

Stellar Evolution

Adapted from `http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php`

Introduction

Stars spend 90% of their lifetime on the Main Sequence, but in the last 10% of their lives, their life tracks take them to other parts of the Hertzsprung-Russell Diagram. The stars change dramatically in temperature, luminosity, and size. They also produce some fascinating end-products in these late evolutionary stages.

Theories of stellar evolution were originally founded on a few optical observations and some physics and chemistry. These initial models were difficult to confirm when they were first developed. However, modern observations, like the discovery of a supernova in the Large Magellanic cloud in 1987, have advanced our understanding of stellar evolution into a true scientific theory. Our understanding comes from a mixture of images at many different wavelengths as well as data, such as light curves. The Hertzsprung-Russell (H-R) diagram is a useful tool for organizing our information. We can trace the changes in luminosity and surface temperature over time on an H-R diagram. In this lab you will look at a number of pictures, including multi-wavelength images and illustrations. Your goal will be to identify the images and put them in place on an H-R diagram.

You can get a pdf with the images here:

`http://www.astro.lsa.umich.edu/Academics/Undergrad/Labs/StelEv/images.pdf`

Images and the H-R Diagram

You have two H-R diagrams, one labeled "High Mass" (50 Msun) and one labeled "Medium Mass" (2 Msun). An evolutionary track is drawn on each, with several points labeled.

Begin by identifying each of the pictures.

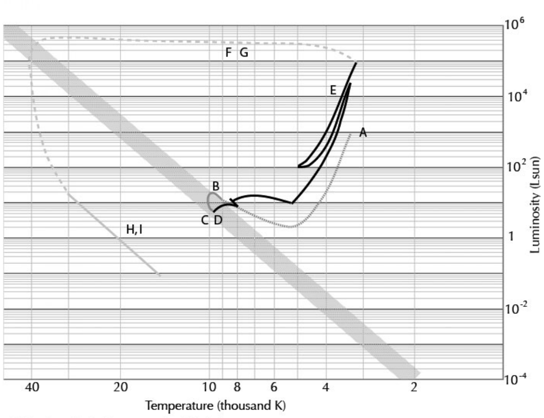
Construct a table as follows:

#	Letter(s)	Description

Write a concise description in your table in the row with the picture number.

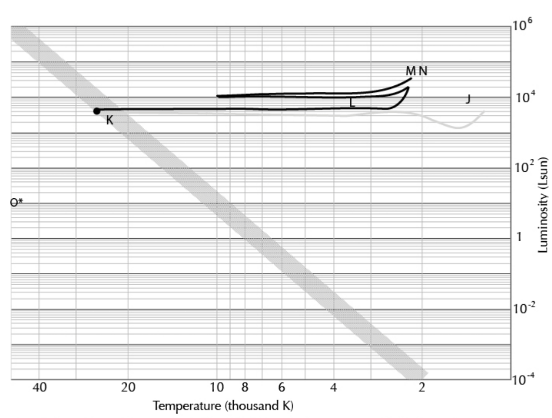
Identify which image belongs with each label on the H-R diagram. Record the letter in table 1. Make sure your description in table 1 makes it clear why the image belongs in that position. All the letters must be used, and places with two letters indicate two images. Some of the images may be used for both high and medium mass stars.

H-R Diagram for Medium Mass Star



Darker dotted line is the proto-star (pre H fusion)
Light dashed line indicates the star is not actually visible
Letters mark general areas on the line.

H-R Diagram for High Mass Star



Events at specific points are marked with a dot. Otherwise, they correspond to that general area on the line.
* This object's position varies greatly (enough so it isn't usually plotted on an HR diagram)

Stellar Structure

Adapted from `http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php`

Introduction

Stars live on the main sequence for astonishingly long times. This implies that internal pressure and gravity are very nearly equal at all points in the star, and that this balance is highly stable, a state known as hydrostatic equilibrium.

Stars begin as large, diffuse gas clouds. These clouds have sizes of parsecs, densities of about 10^3/cc, and temperatures of 10 - 20 K. These clouds are not stable on long timescales, and so easily collapse into fragments under their own gravity. As a fragment continues to shrink under gravity, the gas becomes compressed. A well-known property of a gas is that it heats up when it is compressed, so we know the gas becomes hotter. Furthermore, the gas obeys the equation of state (also known as the ideal gas law):

$$P \propto \rho\, T \tag{15}$$

where P is the gas pressure, T is the temperature, and ρ is the gas density, which is equal to the number of particles per volume. Thus, as both the density and the temperature increase, so does the pressure. Thus, at a certain point, pressure will become equal to gravity, and the fragment/star will not contract any further. By the time fusion starts in the stellar core, the density is about 90 g/cm^3 (15 times denser than steel) and the temperature is about 15 million K.

Fusion takes place in a star's core, but the energy generated must travel to the surface to be visible. There are two basic mechanisms for energy transport: convection and radiation. In *convection*, hot *matter* moves upwards towards the surface, cools, and sinks back down. Radiation, or transport in the form of photons, can occur if the gas of the star is sufficiently transparent. In the Sun, the core is radiative while the outer layers are convective. On Earth, radiation from the hot ground can often escape into space by simply radiating away. But sometimes, this energy causes convection to set up resulting in cumulus and cumulonimbus clouds, especially in the summer. Thus, even in our atmosphere, both mechanisms can operate.

The energy from a mature star derives from the steady thermonuclear fusion of smaller atoms into larger ones. Main sequence stars convert hydrogen into helium. Energy is released due to the small difference in mass, following $E = mc^2$. The equation of state and the hydrostatic equilibrium condition ensure that the core is both sufficiently dense and hot for this fusion to occur. If the density is too low, atoms dont encounter one another frequently enough. If the temperature is too low, the atoms will not fuse. Thus, an understanding of fusion requires not only knowing how much mass is converted to energy, but also how frequently the reaction occurs. This demands knowledge of what is known as the cross section for fusion, or the probability that two atoms will fuse as a function of temperature and density.

The Equation of State

In this section, you will compress a gas to determine the change in temperature and density.

1. Get one of the pistons and note the temperature ($T_{\rm room}$). Convert to Kelvin. Measure the distance from the piston's initial position to the end of the cylinder (h_0). Construct a table as follows.

Trial	Temp (K)	Height

2. Rapidly compress the piston. Wait five seconds, then record the temperature and height as in step 1. Let the piston come back out to the initial position and the temperature drop to room temperature. Repeat thrice. Find the average temperature and height for the four trials. Find the "standard deviation of the mean," given by

$$\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})}{N(N-1)}} \,. \tag{16}$$

   This is your uncertainty in the mean temperature or height.

3. Construct a logical argument showing $\rho \propto 1/h$. Use this information for the next step.

4. Use the ideal gas law to determine by what factor the pressure increased when you compressed the cylinder, P/P_{room}. As always, show your work. Use error propagation to determine the uncertainty in your answer.

5. The density of the Sun's core is 105 times that of air at sea level on Earth, and the temperature is 15 million K. What is the ratio of the pressure at the center of the Sun to that of the Earth at sea level if the temperature on Earth is about 300 K?

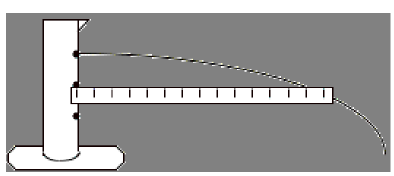
Hydrostatic Equilibrium

We demonstrate that the pressure varies with the force of the weight of water.

- There are 3 holes in the cylinder. Make sure they are plugged and place the cylinder in one end of the tray with the holes facing the center so the tray will catch the water. Fill the cylinder up to the 1000 ml level. Construct a table as follows:

hole	pressure (low, etc.)	$x \pm \delta x$	$y \pm \delta y$	$h \pm \delta h$	$v \pm \delta v$
top					
middle					
bottom					

- Remove the top stopper and, by placing a finger in the stream, subjectively evaluate how much pressure the stream of water exerts. Note this in your table.



- Place a ruler at the next highest hole and measure the distance from the center of the cylinder to the stream, x (see figure). Measure the distance of the hole from the ruler, y. Measure the height of the hole from the bottom of the cylinder, h. For each measurement, estimate your uncertainty.
- Plug the hole and refill the cylinder. Unplug the middle hole and repeat.
- Plug the hole and refill the cylinder. Unplug the bottom hole and repeat.

6. Based on your observations, how does the pressure change with the height in the cylinder?

7. Calculate the speed of each stream as it exited the side of the cylinder:

$$v = \frac{x}{\sqrt{2y/g}}\,. \tag{17}$$

Enter this in your table.

8. Use the above equation and your error propagation handout to determine a formula for δv. Use your formula to calculate δv.

9. Create two plots: x vs. h and v vs. h, including error bars. Why do the two plots have a different shape?

10. Based on your graphs, and given the observed relationship between height and pressure, which of these quantities, x or v, is more closely related to the pressure of the water at each hole? Explain.

Transfer of Energy

- Fill a glass beaker with water and place the beaker on one of the burners of the hot plate. Then add 10-20 glass beads to the water. Raise the temperature until the water begins to boil, then back off until the boiling just stops (about to the **MED** level on the dials). Give the hot plate and water time to adjust.

11. Add ONE drop of yellow food coloring. Count how many beads reach the top of the water (n) over 15 seconds. Do this four times to ensure the temperature is constant (n doesnt vary more than $\sqrt{n}$). Once you have 4 fairly stable counts in a row, record $n = n_1 + n_2 + n_3 + n_4$, and $\delta n = \sqrt{n}$.

12. Add about 10 drops of **blue** food coloring until the water is opaque. Repeat the previous step. Did the number of beads hitting the surface increase or decrease? Is this a significant change, taking δn into account?

13. Which configuration, clear or colored, do you think is more conducive to transferring energy via radiation? Explain.

14. If radiation is suppressed, how else would the energy escape? Did you see evidence for this after adding the dye?

15. Based on your answers to the previous questions, do you think radiation or convection should be dominant in the Sun? Look around for a picture of the Sun to help you address this question, and explain your answer.

Energy Generation

We will investigate the role of the cross section using clay nuclei propelled at a target.

The target consists of a poster board with sticky regions that, if hit by the clay at sufficient speed, have a decent chance of sticking to the board. There are two targets with different densities of target nuclei. You should use both.

- Produce 20-30 nuclei from clay of a single color. Construct a table as follows:

Target	low temp		high temp		high temp probability	low temp probability
	drops	hits	drops	hits		
high density						
low density						

- Lay the target on the ground. Drop nuclei onto it from about knee-high, counting the drops you make. Remove all nuclei that are not on a sticky pad, and all but one if there are multiple on a sticky pad (only one fusion reaction is allowed per pad). Gently lift the board. Count the nucleus as a 'hit' if it remains stuck to one of the pads (watch out for ones that roll off and take out another one on the way.) Record the number of drops and hits in your table.
- Repeat, dropping the nuclei from waist-high.
- Repeat for the other target board.
- The number of hits divided by the number of tosses per trial is the interaction probability, which is closely related to the cross section for fusion. Record the probabilities in your table
- Measure the approximate size of one of the small target squares and calculate the area.

16. Under what conditions was fusion most likely to occur?
17. Lets say you fire a nucleus at the high density target once every minute. Based on your fusion probabilities, how long do you have to wait for a fusion reaction?

Note how knowing the fusion probability allows you to calculate the rate of fusion reactions if you know the temperature and density. In a star, we know the latter from the hydrostatic equation and equation of state, so if we can measure the fusion probability, we can determine the typical time a given atom has to wait to fuse with another nucleus. Consequently, experiments like the one done in this demonstration (though a bit more sophisticated and involving actual H atoms and not clay!) are needed to determine the rate that stars form their energy from fusion.

You can roughly calculate the real interaction cross section for H fusion in the sun by noting that the Sun's luminosity is 4×10^{33} ergs. Fusing H into He converts 0.8% of the mass of the solar core into energy. The core itself is 10% of the solar mass, so $0.008 \cdot 0.1 \cdot 2 \times 10^{33}$ g $= 1.6 \times 10^{30}$ g gets converted into energy. Putting this mass into Einsteins equation yields 1.4×10^{51} ergs. In one second, 2.9×10^{-18} of this total energy is emitted (luminosity divided by the total energy), so 2.9×10^{-18} of the total hydrogen is consumed every second. This is the interaction probability (technically, divided by four since four atoms have to be fused to make one He atom).

To compare this to the experiment you did, you would get a similar interaction rate if the area of one of the little target squares divided by the area of the entire target board was equal to 2.9×10^{-18}!

18. Using the value of the size of the squares, calculate the area of the target board required had we accurately simulated the interaction probability.
19. If the resulting (very large) target board was square, how large would it be one a side? (From this you might appreciate that, as far as the Hydrogen atoms are concerned, the center of the Sun is not such a violent place; collisions strong enough for fusion are in fact extremely rare. Good thing there is a LOT of Hydrogen there.)

The Crab Nebula

Adapted from `http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php`

Introduction

Today, we will study the famous Crab nebula, which is in fact the remnants of the bright supernova of 1054. This supernova was recorded by Chinese astronomers to have been visible during the day for 23 days and in the nighttime sky for two years. In 1968, radio astronomers Staelin and Reifenstein found the neutron star at the core of the nebula. The star's magnetic field causes it to emit beams of light from its magnetic poles. As these beams sweep by, the neutron star appears to blink on and off, and so we call it a "pulsar."

Part I: Finding the Crab Nebula's Age

For this part of the lab, you will need the photographs taken of the Crab nebula in 1973 and 2000 so that you can find the rate of expansion.

1. You must first obtain the scale for each photograph. Measure the distance between stars A and B, estimating to the nearest 0.1mm. Knowing that the angular distance between the stars is 385 arcseconds, find the scale of each photo in units of [arcsec/mm].

Date	Distance between Marked Stars (mm)	Photographic Scale (arcsec/mm)

2. Find the location of the pulsar on the photos, marked by a four-point star. Construct a table as follows:

Knot	Grp	r ('73)	θ ('73)	r ('00)	θ ('00)	$d\theta$	ω ("/yr)	ΔT (yr)
1	1	...	...	...	...	...	...	...
1	2	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...

Four structures in the Crab nebular are enumerated on the photographs. Use a millimeter ruler to measure the distance of each knot, to the nearest 0.1 mm, from the pulsar on both photos. Convert to arcsec using the scale from step 1.

3. For each knot, calculate the average speed of the ejected material in [arsec/yr] using the following formula for angular velocity:

$$w = d\theta/dt \tag{18}$$

4. Now, solve for the total time since the explosion using the simple relation

$$\Delta T = \theta/w\,. \tag{19}$$

Find the estimated expansion time for each knot. Write your four values for ΔT on the chalkboard, along with your group number.

5. Finally, we measure ΔT by taking the average of all values. Furthermore, we determine our uncertainty as follows:

$$\sigma_{\Delta T} = \sqrt{\frac{\sum(\Delta T_i - \bar{\Delta T})^2}{N(N-1)}}\,. \tag{20}$$

6. Compare your value for the date of the supernova event to the accepted value of 1054. What does this suggest about the expansion velocity of the nebula? Explain.

7. In a similar manner, determine $\bar{\omega}$ and its uncertainty.

Part II: The Distance to the Crab Nebula

In its actual motion, v, across the plane of the sky, a knot can be considered as having traversed a tiny fraction of the circumference of the celestial sphere. (The total circumference is $2\pi d$, where d is the distance from the observer to the nebula.) Thus, we can set up a relation between the angular and spactial velocities:

$$w/360° = v/2\pi d \tag{21}$$

We have found the angular rate of expansion, w, of the Crab Nebula. Therefore, if we measure v, we can solve for the distance, d. To accomplish this, we will use the the spectral properties of a supernova remnant and our knowledge of Doppler shifts.

Look at the spectrum of the Crab Nebula. In this negative image, the bright emission lines of the nebula and laboratory comparison spectra above and below show as dark lines. The spectrum is produced by the bright filament of the nebula. Notice that each of the filaments is either red-shifted or blue-shifted, with nothing in between. This occurs because we are seeing material that is either at the very near side of the nebula, rushing towards us, or material at the back side of the nebula, rushing away. The filaments are on the outer edges of the nebula.

8. Examine carefully the region around the [OII] 3727 line on the Crab Nebula Spectra. First calculate the spectral scale using the 3690Å and the 3719Å palladium lines in Å/mm.

9. Measure (in mm) the maximum Doppler shift between the blueshifted and redshifted branches of the [OII] 3727 "necklace". Use the scale you found in the previous step to convert to Å.

10. Use the Doppler formula to calculate the relative velocity between the approaching and receding filaments in km/s. Show your calculation.

$$\frac{\Delta\ell}{\ell} = \frac{v}{c} \tag{22}$$

11. What final step must we perform to determine the expansion velocity? Perform it.

12. Use equation 21 to calculate the distance to the Crab Nebula in light years. Show your work, and be careful with units.

13. Use error propagation to calculate your uncertainty in the distance.

Thinking Questions

14. When calculating the date of the supernova explosion, what did you assume about the velocity of the gaseous knots?

15. When calculating the spatial velocity of expansion in km/s and the distance to the Crab Nebula, what have you assumed about the shape of the nebula?

16. With the high resolution of radio telescopes, namely the Very Long Baseline Interferometer, you can use the method of expansion parallax for supernova remnants that are much farther away from us than the Crab Nebula. Supernova 1987A exploded in the large Magellanic Cloud and was first observed on February 23, 1987. At that point, the remnant had a radius of zero. 5.2 days later, the remnant had a radius of 0.0022". The radial velocity of the nebula was 36,000 km/s. What distance does this suggest to the Large Magellanic Cloud?

Standard Deviation of a Data Set

Given a set of data $\{x_1, x_2, x_3, ..., x_N\}$, we often wish to describe the types of values we encounter in such a data set. A good way to due this is using the mean, standard deviation, and the standard deviation of the mean.

The mean, which you are undoubtedly already familiar with, is given by

$$\bar{x} = \frac{1}{N} \sum x_i \,. \tag{23}$$

The standard deviation is a quantity that describes the spread in the data set from the mean. It is given by

$$s = \sqrt{\frac{1}{N-1} \sum (x_i - \bar{x})^2} \,. \tag{24}$$

We may also wish to express how certain we are about our mean. If we were to repeat our experiment, we would obtain different data, and thus would obtain a different mean value. A measure of about how much we would expect the mean to change is given by the standard deviation of the mean, or standard error:

$$\sigma = \sqrt{\frac{1}{N(N-1)} \sum (x_i - \bar{x})^2} \,. \tag{25}$$

Galactic Star Clusters

Introduction

A color-magnitude diagram is a plot of apparent magnitude versus the (B-V) color index of a group of stars. An H-R diagram is usually a plot of absolute magnitude versus spectral type or color index. Note that these are equivalent: a plot of brightness versus stellar temperature.

Clusters (close spatial groups of stars) form from a single clump of gas at a particular time. Thus, we can safely assume that the stars in a cluster have roughly the same distance (and thus distance modulus), age, and initial chemical composition. As a result, the location of a cluster star in the color-magnitude diagram relative to its companions is due primarily to its mass. Astronomers can then use this information to check theories of how stars evolve, i.e. change over time.

For example, we know that low mass stars live longer than high mass stars. If this is correct, then a very old cluster should contain only low mass main sequence stars. Furthermore, low mass stars also form much more slowly than high mass stars. Thus, a very young cluster should contain more high mass main sequence stars, because the low mass stars haven't made it onto the main sequence yet.

Equipment

Open Internet Explorer and point the browser to the following URL:

`http://www.astro.lsa.umich.edu/~grad/clusters`

The program will begin loading a set of color-magnitude diagrams for eight clusters in our galaxy. Notice that the diagram plots apparent magnitude, V, vs. color index, (B-V). However, along the top x-axis is a quantity labeled $(B\text{-}V)_0$, which is the color index corrected for interstellar reddening (we'll get to that later). Color index is a measure of the color of a star. The smaller the value of (B-V), the bluer the color of the star.

Choose a cluster and click somewhere on the plot. You should see blue cross-hairs. Values corresponding to the cross-hairs are printed in the lower right.

Finally, turn the ZAMS on with the menu in the upper right. This will produce a red gridded overlay; identify the ZAMS line and compare the red axes with those of the color-magnitude diagram underneath. Moving the slider bars will allow you to slide the ZAMS around; clicking the arrows will allow finer adjustments. Use the menu to turn the ZAMS on and off.

Turn-off Point and Age

We can determine the age of a cluster fairly accurately using the turn-off point, the spot where stars begin to deviate from the main sequence. Below the turn-off, stars are still burning hydrogen in their cores, happily living on the main sequence. Above the turn-off point, stars have exhausted their core hydrogen. For a younger cluster, the turn-off point is closer to the blue (high mass, bright) end of the main sequence, and for older clusters, it is closer to the red end of the main sequence.

Construct a table as follows:

Cluster	$(B\text{-}V)_0$	Cluster Age	$(V\text{-}M_V)$	Distance (pc)	$E(B\text{-}V)$
M67					
...					

To accurately locate the turn-off point, use the ZAMS overlay. First, match up the *upper* x-axis of the *overlay* and the upper x-axis of the *diagram* at $(B\text{-}V)_0 = 0.0$ by using the horizontal slider. Then slide the overlay up/down with the vertical slider until you get what you consider to be the best match between the star data points and the ZAMS line. When fitting clusters with a lot of scatter, try to match the narrower parts of scatter to the curve, and generally try to keep the ZAMS to the lower left of the scatter since objects not on the main sequence are probably above and right of the ZAMS. When you've got a match, you will be able to see where the star data "peels off" from the ZAMS – this is the turn-off point. Use the cross-hairs to measure $(B\text{-}V)_0$, and record this in your table.

Use the graph of cluster age vs. turn-off point color included in this lab by matching your $(B\text{-}V)_0$ to the value on the x-axis and reading the corresponding age off the y-axis. Record the age in your table, estimating your uncertainty.

Distance Modulus

The distance modulus is the quantity $(m\text{-}M)$: the difference between the apparent and absolute magnitudes. We measure this vertical offset between the cluster stars (m) and a set of standard stars (M), by sliding the standard stars (the ZAMS) up or down until they line up with the cluster stars. To find the distance modulus, keep your ZAMS overlay lined up as before so the ZAMS matches the star data points. Clicking the crosshairs at any point will measure V from the cluster color-magnitude diagram and M_V from the ZAMS overlay. Subtract to get $(V\text{-}M_V)$. (This is even easier if you click where $M_V = 0$.) Record the distance modulus in your table, recording your uncertainty.

To convert this into the distance of the cluster, we must invert the distance modulus equations.

$$(V\text{-}M_V) = -5\ \log(d/10\text{ pc}) \qquad (26)$$

$$d = (10\text{ pc}) \times 10^{(V-M_V)/5} \qquad (27)$$

Record the distance in your table. Use error propagation to determine your uncertainty.

Effects of Interstellar Dust

You may have noticed that the sun or bright full moon can appear orange or even red at sunset or sunrise. This is because you are looking through a lot of dust in the air at those angles. Dust tends to absorb and selectively scatter blue light more effectively than red. The red light can pass by the dust grains more easily than the blue, so we end up receiving a higher fraction of the red light that is emitted than of the blue. This applies to dust in our atmosphere (resulting in a redder, dimmer sun at sunset) and also applies to dust between the stars (resulting in much the same thing).

So two things occur due to interstellar dust: reddening (redder light) and extinction (less light).

Reddening

Our observed color index $(B\text{-}V)$ may be larger (redder) than the true color index of the stars due to the reddening effect of the dust. We estimate how much reddening has occurred by using the spectral type to calculate an intrinsic, un-reddened color index. This is labeled $(B\text{-}V)_0$. The difference between the observed color index $(B\text{-}V)$ and the intrinsic color index $(B\text{-}V)_0$ is called the color excess, or $E(B\text{-}V)$, of the star. This is defined as:

$$E(B\text{-}V) = (B\text{-}V) - (B\text{-}V)_0 \qquad (28)$$

If you look at the upper and lower x-axes on the cluster color-magnitude diagrams, you will notice that in each case the $(B\text{-}V)$ and $(B\text{-}V)_0$ scales are offset such that the $(B\text{-}V)_0$ scale is shifted to the right with respect to the lower scale (to the right corresponds to a lower value of the color index, and thus is bluer). This should make sense in light of the reddening effects of dust. Measure the color excess for each diagram by using the cross-hairs to measure $(B\text{-}V)_0$ and $(B\text{-}V)$ at some point in the plot. Once again, a little math should give you $E(B\text{-}V)$. (And once again, clicking where $(B\text{-}V)_0 = 0$ will make this trivial.) Record the color excess in your table.

Extinction

Stars also appear dimmer due to interstellar dust. This is called extinction. Extinction affects our measurement of a star's brightness and therefore our determination of the star's distance if we use the distance modulous equation. Let's find out how extinction has affected our data.

Construct a table as follows:

Cluster	True Distance, d (pc)	Measured Distance, d' (pc)	d'/d
M67	800		
...			

Obtain the true distances to the star clusters from your GSI. Rewrite your distances in the column marked d' (your measured distance) and calculate the ratio d'/d for the third column. Notice the errors.

1. Draw a sketch, with a dust cloud and several rays of light (blue light and red light). Include the effects the dust has on the light. Explain what else, besides reddening, could happen to the appearance of a cluster because of dust.
2. Plot the ratio d'/d versus $E(B\text{-}V)$. Describe the trend you see in your data. Carefully and logically explain the physical reasons for this trend.
3. Suppose you have two clusters that are both 1000 pc away, but are in different constellations. One of the clusters has $E(B\text{-}V)= 0.6$, while the other has a color excess of 0. What does this tell you about the distribution of dust in the Galaxy?
4. Draw a sketch of a color-magnitude diagram, and indicate which regions in the chart represent red/blue, bright/dim, and cool/hot stars. What kind of stars live in each region?
5. Cluster Mel 20 contains objects in the lower right of the CMD that are not on the main sequence. These are not evolved red giant stars – how can you tell? What stage of stellar evolution do they represent?
6. While there was quite a bit of variation in the clusters we looked at, none of them was as old as a globular cluster (these were galactic clusters). If you were to look at a color-magnitude diagram of a very, very old cluster, what would you expect to see? Draw a sketch as well as explain any new features.
7. Could you use the technique we have used here to find the age of a single star (not in a cluster)? Why/why not? In general, can you tell anything about the age of a single star from an H-R diagram?

General Error Propagation Formula

By this time, you have had some practice applying error propagation formulae to the basic mathematical operations: addition, subtraction, multiplication, division, and exponents. However, it is sometimes necessary to determine the error in a calculation resulting from other functions, such as logarithmic or trigonometric functions.

There is always a simple method for calculating error in such cases:

- Say you are given a function $f(x)$, and you have measured x and δx.
- Then, $\delta f = (f(x + \delta x) - f(x - \delta x))/2$.

While this method works, it can be time-consuming, particularly if there is more than one independent variable. Thus, we then choose to use the following formula:

$$\delta f = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i} \delta x_i \right)^2} \tag{29}$$

You can easily verify that this formula reproduces Eqs. 10, 12, and 14.

For further reading on this topic, see *Data Reduction and Error Analysis for the Physical Sciences*, by Philip R. Bevington and D. Keith Robinson.

Cepheid Variables

Introduction

Trigonometric parallax works only for stars very close to us. Spectroscopic parallax, the method used in the galactic star clusters lab, allows us to measure distances to any star cluster for which we can graph the main sequence. However, red main sequence stars are very faint, making it nearly impossible to resolve them even in the galaxies closest to us. Thus we must use more indirect means to determine how far away other galaxies are. One such tool involves the use of Cepheid Variable Stars.

With time, virtually all variable stars have proven themselves to be particularly useful stars. Eclipsing binaries reveal stellar masses; novae inform us about mass transfer betwen stars; supernovae reveal details of the end stages of very massive stars; Mira variables reflect the radical changes happening deep within red giants. And virtually all stars have proven to be useful distance indicators. Cepheids are particularly useful because they are luminous (they correspond to supergiant stages of the lives of fairly high mass stars) and their brightness varies characteristically, making them easy to find even in very distant galaxies.

The light curve of a variable star is a plot of the star's brightness over time. Plotting the light curves of Cepheid variables reveals an important property, the period-luminosity (P-L) relationship:

$$\log\left(\frac{L}{L_\odot}\right) = 1.5 \cdot \log\left(\frac{P}{1 \text{ day}}\right) + 1.3 \tag{30}$$

where L is the star's average luminosity, and P is the variable's period. Cepheids typically have periods ranging from 3 to 100 days.

One can thus use the P-L relationship to determine the luminosity of the star. We then combine luminosity and flux to determine the star's distance:

$$f = \frac{L}{4\pi d^2} \tag{31}$$

where L is the luminosity, f is the flux (apparent brightness) and d is the distance.

In this lab you will have to identify several cepheids from a field of stars. Your GSI will then provide you with data for those stars so you can generate their light curves and find the distance to their host galaxy. This is best done in groups of 3 - 4.

Identifying Variable Stars

Get a pair of images from your GSI. These images depict a small region of a galaxy with six simulated variable stars added. The image pair corresponds to observations of the galaxy at different times.

1. After taking a quick look at the images, figure out what to look for to find the variable stars. Describe what you will look for and how you will identify the variables:

2. There are no fewer than six Cepheid variables in the field. Finding the sixth is quite tricky. This search should give you a taste of what it is like to look for variables in actual data, although you've been given "nice" data (fake variables with nice periods were added to an actual image). Imagine there were 3 variable stars in the field with periods of 5, 10 and 20 days, and the two images were taken 10 days apart. How many variables would you be able to identify? Why?

3. Once you have identified at least four variables, show your GSI where you think they are. He/she will record how many stars you correctly identified here:

Light Curves

Your GSI will give you data tables for stars in 4 different galaxies. They plot flux vs. date (in days since the beginning of the observation.)

1. Plot the flux vs. date for each of the three stars in galaxy 1 (3 graphs). Why are there gaps in the dates in the plots?
2. Is the light curve of a Cepheid variable a simple sine curve?
3. Explain how to determine the period and mean flux from the light curve using a sketch of a Cepheid light curve.
4. Construct a table as follows:

Galaxy	Star	P (days)	f_{ave}
1	1		
1	2		
1	3		
2	1		
...	...		

5. Complete your table using the light curves for the stars in galaxies 1 - 4. Estimate your uncertainty in f.

The Distance Modulus

1. Using Eqs. 30 and 31, derive a formula expressing $\log f$ in terms of $\log P$ and d.
2. Plot $\log f$ vs. $\log P$ for the three stars in galaxy 1. Determine the slope and intercept of the best-fit line, as well as the uncertainty in these values, using Eqs. 2 - 6. I recommend showing your work by making your own functions in Excel and putting these functions into your report.
3. Repeat this process for each galaxy.
4. What expected value does the slope correspond to? What does the intercept tell you?
5. Should the intercept and slope be exactly the same, similar, or not the same for each galaxy? Explain.
6. Transform your numbers into a distance, in parsecs, to each galaxy. Use error propagation (see page 27) to determine the uncertainty in the distance. Compare the distances, taking your calculated uncertainty into account.

Thinking Questions

1. In this lab, you had to determine the luminosity. Based on what you did, explain why the brightness, or flux, is generally more accurate than the luminosity.
2. Lets say we can measure Cepheids out to a flux of 2×10^{-47} W/m^2. Using the P-L and inverse-square distance relations, what is the furthest galaxy we could detect Cepheids in if we limit ourselves to a maximum period of 100 days?

The Tully-Fisher Relation

Introduction

We know that the Sun is moving in a circle around the center of the Milky Way, constantly accelerated by the gravitational pull of our galaxy. In fact, all material in the galaxy does the same, producing a spinning disk. We notice likewise that all other spiral galaxies also appear to have spinning disks: when we point the spectrograph at one side of the disk, the spectral lines are all Doppler-shifted towards the red, meaning that the stars are receding from us. On the other side of the disk, the stars seem to be moving towards us, so we conclude that the disk is spinning. Gravity is once again at work, keeping the stars in these disks from flying off. We will use this Doppler shift to measure how fast several spiral galaxies are spinning. The faster the disk spins, the stronger gravity must be. By measuring this gravity, we can determine the mass of the spiral galaxy, since Newton's Law of Universal Gravitation shows that gravity is proportional to mass. We can also measure the galaxy's luminosity, and, using the average luminosity of a star, we can estimate the number of stars in each spiral galaxy. Then we can ask whether there is more mass in the galaxy than can be accounted for by stars (i.e., what is the mass-to-light ratio?). As we shall see, there must be some kind of dark matter whose gravity keeps the spiral galaxy together, but which does not appear to shine like stars do.

Along the way to discovering dark matter, you'll notice an amazing correlation between the rotation speed and luminosity of a spiral galaxy, known as the Tully-Fisher relation. This relation enables us to use spiral galaxies as standard candles, similar to Cepheid stars but millions of times brighter. By comparing your spiral galaxies to our next-door neighbor spiral galaxy (the Andromeda galaxy), you'll be able to figure out how far away they are, and measure the Hubble constant and the age of the Universe!

Measuring the Galaxies' Rotation

Your instructor will give you a set of rotation curves for six very distant spiral galaxies along with CCD photographs of these six galaxies (data taken by Prof. Bernstein). Look at the pictures of the six galaxies. Note that you cannot see the spiral arms because the galaxies are nearly edge-on and because the pictures are over-exposed. Also note that some are bigger or brighter than others. The apparent magnitude, m, of each galaxy has already been determined and is given in Table 1.

Galaxy	1	2	3	4	5	6
m	11.75	11.48	11.51	13.25	12.61	12.47

Table 1: Apparent magnitudes for spiral galaxies.

The rotation curves plot the observed redshift for the H-alpha line at various positions along the disk of each galaxy. Remember that H-alpha normally has a wavelength of 656.3 nm, so these galaxies must all be redshifted considerably. In addition to this overall redshift, notice that the wavelength for one side of the galaxy is longer than for the other side, indicating that that side is moving away from us faster than the other side. The most obvious explanation for this is that these galaxies are spinning, just like our own Milky Way.

You can use the Doppler shift formula to calculate how fast each half of the galaxy is moving away from us. You should recall that the Doppler shift formula gives us the recessional velocity as:

$$v = \frac{\lambda - \lambda_0}{\lambda_0} c \qquad (32)$$

As usual, c is the speed of light, and $\lambda_0 = 656.3$ nm.

- Construct a table as follows:

Galaxy	m	$v_{\rm left}$	$v_{\rm right}$	$v_{\rm rot}$	$\log_{10}(v_{\rm rot})$
1	11.75	...			

- For each galaxy, determine the typical wavelength for the Hydrogen line on both sides of the disk. Do not forget to estimate your uncertainty. Record these values in your table

- Use Eq. 32 to change these wavelengths into speeds (in km/s).

1. Again using Eq. 32, derive an expression for the uncertainty in v in terms of the uncertainty in λ. Remember to show your work! Use your formula to calculate δv.

- Even the "approaching" side of the disk appears to be moving away from Earth at several thousand km/s. This is due to the expansion of the Universe, which causes the entire galaxy to rush away from us at some expansion (or recession) velocity, $v_{\rm exp}$. If the rotation speed is $v_{\rm rot}$, then the "approaching" side will show a Doppler recession velocity of $v_{\rm exp} - v_{\rm rot}$, and the receding side will be $v_{\rm exp} + v_{\rm rot}$. The difference in these values is $2 \cdot v_{\rm rot}$. So in the column labeled $v_{\rm rot}$, you should enter half the difference between the left and the right side. Then calculate the log of each of these numbers for the last column.

2. Make a plot of m vs. $\log_{10} v_{\rm rot}$. Comment on the apparent relation between m and $\log_{10} v_{\rm rot}$.

3. Construct a best-fit line, and determine its slope and intercept, as well as the uncertainty in these values, using Eqs. 2 - 6. I recommend showing your work by making your own functions in Excel and putting these functions into your report.

This line relates the rotation speed of a galaxy to its magnitude, and is the so-called Tully-Fisher Relation.

Distance Measurements and the Hubble Constant

The six galaxies you have been studying are all (roughly) the same distance away. We know this because the rotation curves all straddle the same wavelength, (roughly) 671.5 nm. This means they are all moving away from us at approximately the same speed. According to the Hubble Law, the recessional velocity is proportional to a galaxy's distance, so these galaxies must all be around the same distance from us.

Now you are going to figure out how far away this is using your determined Tully-Fisher relation. Your plot should show a correlation between a galaxy's rotational velocity and its apparent magnitude. To convert this into information about the galaxies' absolute magnitude, or luminosity, we must calibrate the relation using a galaxy of known distance. Luckily, there is a spiral galaxy, M31, also called the Andromeda galaxy, which is very close by so that we can measure its distance using Cepheid variable stars. M31 is 770 kpc away from us, it has an apparent magnitude of 1.04, and its rotational speed $v_{\rm M31}$ has been observed to be 275 km/s.

First, you should determine how bright M31 would be if it were at the distance of the other six galaxies. To do this, you need to calculate the logarithm of its velocity (275 km/s) and see where this falls on your graph to get the corresponding apparent magnitude. We'll call this magnitude m_d since it is the magnitude M31 would have at the as yet undetermined distance, d.

4. For now, let your version of the Tully-Fisher relation be given simply by $m = a \cdot \log_{10} v_{\rm rot} + b$. Use this to write a formula of the form $m_d = m_d(a, v_{\rm M31}, b)$

M31 is much closer to us than the other six, and its apparent magnitude is seen to be $m = 1.04$. We now calculate d from

$$d = d_{\rm M31} 10^{\Delta m/5} \,, \tag{33}$$

where $\Delta m = m_d - m_{\rm M31}$, and $d_{\rm M31}$ is the observed distance to M31.

5. Put your previous answer together with Eq. 33 to obtain a relation of the form $d = d(d_{\rm M31}, m_{\rm M31}, a, v_{\rm M31}, b)$
6. Use your formula to calculate d.
7. Now, apply your mad error propagation skillz to obtain a relation of the form $\delta d = \delta d(d, d_{\rm M31}, m_{\rm M31}, a, \delta a, v_{\rm M31}, b, \delta b)$
8. Use your formula from #7 to calculate δd.

Now we can compute the Hubble constant, H_0;

$$v_{\rm exp} = H_0 d \tag{34}$$

For each galaxy, $v_{\rm exp}$ is the average of $v_{\rm left}$ and $v_{\rm right}$. Therefore, the average $v_{\rm exp}$ for the group of galaxies is the average of all twelve values of $v_{\rm left}$ and $v_{\rm right}$.

9. Compute the mean ($v_{\rm exp}$) and standard deviation of the mean ($\delta v_{\rm exp}$) of all twelve values of $v_{\rm left}$ and $v_{\rm right}$.
10. Now, compute H_0.
11. Now, apply error propagation to Eq. 34 to obtain δH_0.
12. Convert your answer to units of km/s/Mpc.

For many years, a veritable blood feud has been going on between various astronomers who think that this number is either 50 km/s/Mpc or 100 km/s/Mpc. More recently there seems to be a consensus developing that it is somewhere in between. Where does your value fit in?

The Mass to Light Ratio

Now you can estimate the total mass of your galaxies. Consider a star of mass $M_\bigstar$ that orbits within its host spiral galaxy with speed v at a distance R from the center of the galaxy. It must obey Newton's laws and Kepler's 3rd law; therefore,

$$\frac{F = G M_\bigstar M_{\rm gal}}{R^2 \,.} \tag{35}$$

Because it is moving in a roughly circular orbit, we may also use the expression for a centripetal force:

$$F = \frac{M_\bigstar v^2}{R} \,, \tag{36}$$

where G is the gravitational constant, 6.67×10^{-11} $\rm m^3/s^2kg$.

13. Combine Eqs. 35 and 36 to eliminate $M_\bigstar$ and solve for $M_{\circlearrowleft}$.

- Construct a table as follows:

Galaxy	Diam (mm)	Diam (")	R (kpc)	$M_{\circlearrowleft}$ ($M_\odot$)	Mag, M	L ($L_\odot$)	M/L
1	...						

Using your equation, you should be able to find the galaxy mass from the rotation speed v_{rot} (which we know) and the galaxy size R, which we will determine from the galaxy images.

- For each galaxy, measure the distance from one end to the other (in mm). This will correspond to the diameter of the galaxy. Next measure the length of the calibration line shown under the image for galaxy 5. This gives you the scale for the images. Multiply the scale (in arcsec/mm) by your diameter measurements to get the angular diameter, θ, in arcsec. Record these values in your table.

You can now infer the real size based on the galaxy's distance from us, which you already calculated. To do this, use the small angle formula, which in this case would be:

$$R = \frac{1}{2} \cdot 2\pi d \cdot \frac{\theta}{360^\circ} \tag{37}$$

- Use this formula to determine R for each galaxy, and record these values in your table.

To investigate the possibility of dark matter in these galaxies, you need to know how much light all this mass is producing. To figure out how many Suns' worth of light they are emitting, you need to determine how much brighter the galaxies are than the sun. To do this, you need the absolute magnitudes, M, of these galaxies. Use the formula

$$m - M = -5 \ \log\left(d/10 \text{ pc}\right). \tag{38}$$

This should yield a negative number, since these galaxies are much brighter than individual stars.

- Use Eq. 38 to determine each galaxy's absolute magnitude, and record these values in your table.

Now you can figure out how many Sun's worth of light your galaxy is putting out:

$$\frac{L_{\circlearrowleft}}{L_{\odot}} = 10^{\Delta M/2.5}, \tag{39}$$

where ΔM is the difference between the sun's absolute magnitude ($M_{\odot} = 4.8$) and the galaxy's.

- Record $L_{\circlearrowleft}/L_{\odot}$ in your table.
- Finally, calculate the so-called mass-to-light ratio, M/L.

If a galaxy were made up entirely of stars like the sun, this ratio would be exactly 1. (However, this doesn't mean you will get M/L values near 1 for your galaxies . . .)

Thinking Questions

14. Why do you think we are using spiral galaxies that look nearly edge-on? What kind of Doppler shift do you think you would measure if the spiral galaxy were nearly face-on instead?

15. It was mentioned that the six galaxies in this lab are all nearly the same distance from us. Why is this important? How would your graph have looked if the galaxies were all at different distances?

16. Why does the Tully-Fisher relation make sense? Think about how the brightness of a galaxy relates to its mass, and how its mass relates to it rotational velocity.

17. You should have found M/L ratios significantly greater than 1. Since stars typically have M/L values of around 1, what does this imply about these galaxies. What kind of objects would increase M/L and why?