

Stellar Structure

Adapted from <http://www.astro.lsa.umich.edu/Academics/Undergrad/labs.php>

Introduction

Stars live on the main sequence for astonishingly long times. This implies that internal pressure and gravity are very nearly equal at all points in the star, and that this balance is highly stable, a state known as hydrostatic equilibrium.

Stars begin as large, diffuse gas clouds. These clouds have sizes of parsecs, densities of about $10^3/\text{cc}$, and temperatures of 10 - 20 K. These clouds are not stable on long timescales, and so easily collapse into fragments under their own gravity. As a fragment continues to shrink under gravity, the gas becomes compressed. A well-known property of a gas is that it heats up when it is compressed, so we know the gas becomes hotter. Furthermore, the gas obeys the equation of state (also known as the ideal gas law):

$$P \propto \rho T \tag{15}$$

where P is the gas pressure, T is the temperature, and ρ is the gas density, which is equal to the number of particles per volume. Thus, as both the density and the temperature increase, so does the pressure. Thus, at a certain point, pressure will become equal to gravity, and the fragment/star will not contract any further. By the time fusion starts in the stellar core, the density is about 90 g/cm^3 (15 times denser than steel) and the temperature is about 15 million K.

Fusion takes place in a star's core, but the energy generated must travel to the surface to be visible. There are two basic mechanisms for energy transport: convection and radiation. In *convection*, hot *matter* moves upwards towards the surface, cools, and sinks back down. Radiation, or transport in the form of photons, can occur if the gas of the star is sufficiently transparent. In the Sun, the core is radiative while the outer layers are convective. On Earth, radiation from the hot ground can often escape into space by simply radiating away. But sometimes, this energy causes convection to set up resulting in cumulus and cumulonimbus clouds, especially in the summer. Thus, even in our atmosphere, both mechanisms can operate.

The energy from a mature star derives from the steady thermonuclear fusion of smaller atoms into larger ones. Main sequence stars convert hydrogen into helium. Energy is released due to the small difference in mass, following $E = mc^2$. The equation of state and the hydrostatic equilibrium condition ensure that the core is both sufficiently dense and hot for this fusion to occur. If the density is too low, atoms don't encounter one another frequently enough. If the temperature is too low, the atoms will not fuse. Thus, an understanding of fusion requires not only knowing how much mass is converted to energy, but also how frequently the reaction occurs. This demands knowledge of what is known as the cross section for fusion, or the probability that two atoms will fuse as a function of temperature and density.

The Equation of State

In this section, you will compress a gas to determine the change in temperature and density.

1. Get one of the pistons and note the temperature (T_{Room}). Convert to Kelvin. Measure the distance from the piston's initial position to the end of the cylinder (h_0). Construct a table as follows.

Trial	Temp (K)	Height
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- Rapidly compress the piston. Wait five seconds, then record the temperature and height as in step 1. Let the piston come back out to the initial position and the temperature drop to room temperature. Repeat thrice. Find the average temperature and height for the four trials. Find the “standard deviation of the mean,” given by

$$\sigma_x = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N(N-1)}}. \quad (16)$$

This is your uncertainty in the mean temperature or height.

- Construct a logical argument showing $\rho \propto 1/h$. Use this information for the next step.
- Use the ideal gas law to determine by what factor the pressure increased when you compressed the cylinder, P/P_{ROOM} . As always, show your work. Use error propagation to determine the uncertainty in your answer.
- The density of the Sun’s core is 105 times that of air at sea level on Earth, and the temperature is 15 million K. What is the ratio of the pressure at the center of the Sun to that of the Earth at sea level if the temperature on Earth is about 300 K?

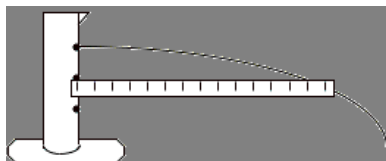
Hydrostatic Equilibrium

We demonstrate that the pressure varies with the force of the weight of water.

- There are 3 holes in the cylinder. Make sure they are plugged and place the cylinder in one end of the tray with the holes facing the center so the tray will catch the water. Fill the cylinder up to the 1000 ml level. Construct a table as follows:

hole	pressure (low, etc.)	$x \pm \delta x$	$y \pm \delta y$	$h \pm \delta h$	$v \pm \delta v$
top					
middle					
bottom					

- Remove the top stopper and, by placing a finger in the stream, subjectively evaluate how much pressure the stream of water exerts. Note this in your table.



- Place a ruler at the next highest hole and measure the distance from the center of the cylinder to the stream, x (see figure). Measure the distance of the hole from the ruler, y . Measure the height of the hole from the bottom of the cylinder, h . For each measurement, estimate your uncertainty.
 - Plug the hole and refill the cylinder. Unplug the middle hole and repeat.
 - Plug the hole and refill the cylinder. Unplug the bottom hole and repeat.
- Based on your observations, how does the pressure change with the height in the cylinder?

7. Calculate the speed of each stream as it exited the side of the cylinder:

$$v = \frac{x}{\sqrt{2y/g}}. \quad (17)$$

Enter this in your table.

8. Use the above equation and your error propagation handout to determine a formula for δv . Use your formula to calculate δv .
9. Create two plots: x vs. h and v vs. h , including error bars. Why do the two plots have a different shape?
10. Based on your graphs, and given the observed relationship between height and pressure, which of these quantities, x or v , is more closely related to the pressure of the water at each hole? Explain.

Transfer of Energy

- Fill a glass beaker with water and place the beaker on one of the burners of the hot plate. Then add 10-20 glass beads to the water. Raise the temperature until the water begins to boil, then back off until the boiling just stops (about to the **MED** level on the dials). Give the hot plate and water time to adjust.
11. Add ONE drop of yellow food coloring. Count how many beads reach the top of the water (n) over 15 seconds. Do this four times to ensure the temperature is constant (n doesn't vary more than \sqrt{n}). Once you have 4 fairly stable counts in a row, record $n = n_1 + n_2 + n_3 + n_4$, and $\delta n = \sqrt{n}$.
 12. Add about 10 drops of **blue** food coloring until the water is opaque. Repeat the previous step. Did the number of beads hitting the surface increase or decrease? Is this a significant change, taking δn into account?
 13. Which configuration, clear or colored, do you think is more conducive to transferring energy via radiation? Explain.
 14. If radiation is suppressed, how else would the energy escape? Did you see evidence for this after adding the dye?
 15. Based on your answers to the previous questions, do you think radiation or convection should be dominant in the Sun? Look around for a picture of the Sun to help you address this question, and explain your answer.

Energy Generation

We will investigate the role of the cross section using clay nuclei propelled at a target.

The target consists of a poster board with sticky regions that, if hit by the clay at sufficient speed, have a decent chance of sticking to the board. There are two targets with different densities of target nuclei. You should use both.

- Produce 20-30 nuclei from clay of a single color. Construct a table as follows:

Target	low temp		high temp		high temp	low temp
	drops	hits	drops	hits	probability	probability
high density						
low density						

- Lay the target on the ground. Drop nuclei onto it from about knee-high, counting the drops you make. Remove all nuclei that are not on a sticky pad, and all but one if there are multiple on a sticky pad (only one fusion reaction is allowed per pad). Gently lift the board. Count the nucleus as a ‘hit’ if it remains stuck to one of the pads (watch out for ones that roll off and take out another one on the way.) Record the number of drops and hits in your table.
- Repeat, dropping the nuclei from waist-high.
- Repeat for the other target board.
- The number of hits divided by the number of tosses per trial is the interaction probability, which is closely related to the cross section for fusion. Record the probabilities in your table
- Measure the approximate size of one of the small target squares and calculate the area.

16. Under what conditions was fusion most likely to occur?

17. Lets say you fire a nucleus at the high density target once every minute. Based on your fusion probabilities, how long do you have to wait for a fusion reaction?

Note how knowing the fusion probability allows you to calculate the rate of fusion reactions if you know the temperature and density. In a star, we know the latter from the hydrostatic equation and equation of state, so if we can measure the fusion probability, we can determine the typical time a given atom has to wait to fuse with another nucleus. Consequently, experiments like the one done in this demonstration (though a bit more sophisticated and involving actual H atoms and not clay!) are needed to determine the rate that stars form their energy from fusion.

You can roughly calculate the real interaction cross section for H fusion in the sun by noting that the Sun’s luminosity is 4×10^{33} ergs. Fusing H into He converts 0.8% of the mass of the solar core into energy. The core itself is 10% of the solar mass, so $0.008 \cdot 0.1 \cdot 2 \times 10^{33} \text{ g} = 1.6 \times 10^{30} \text{ g}$ gets converted into energy. Putting this mass into Einsteins equation yields 1.4×10^{51} ergs. In one second, 2.9×10^{-18} of this total energy is emitted (luminosity divided by the total energy), so 2.9×10^{-18} of the total hydrogen is consumed every second. This is the interaction probability (technically, divided by four since four atoms have to be fused to make one He atom).

To compare this to the experiment you did, you would get a similar interaction rate if the area of one of the little target squares divided by the area of the entire target board was equal to 2.9×10^{-18} !

18. Using the value of the size of the squares, calculate the area of the target board required had we accurately simulated the interaction probability.
19. If the resulting (very large) target board was square, how large would it be one a side? (From this you might appreciate that, as far as the Hydrogen atoms are concerned, the center of the Sun is not such a violent place; collisions strong enough for fusion are in fact extremely rare. Good thing there is a LOT of Hydrogen there.)