Topic B

"The New Astronomy"

Part 3. Kepler's Laws 2 & 3

(Web Version: 10-01-01)

Kepler's Second Law

Kepler's Second Law

- Alternate Name — ***Law of Areas***
- Common Textbook Statement . . .

"The line from the Sun to a planet sweeps out equal areas in equal times"

Summary

Equal ***areas*** in equal ***times***

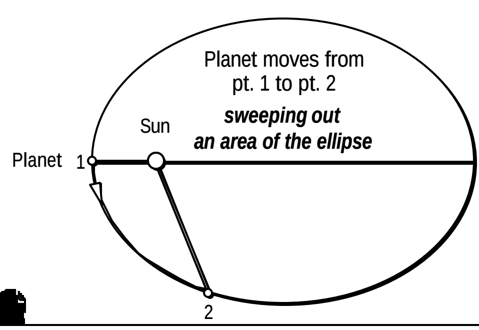
which means

Areas proportional to times

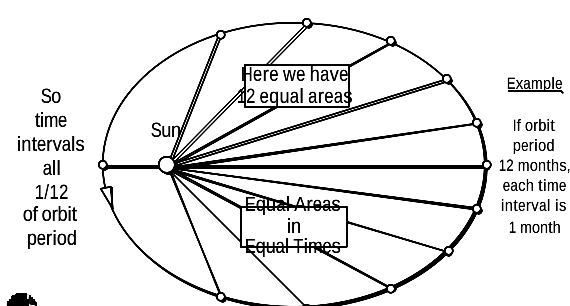
Areas **µ** Times

Note: **µ** means "proportional to"

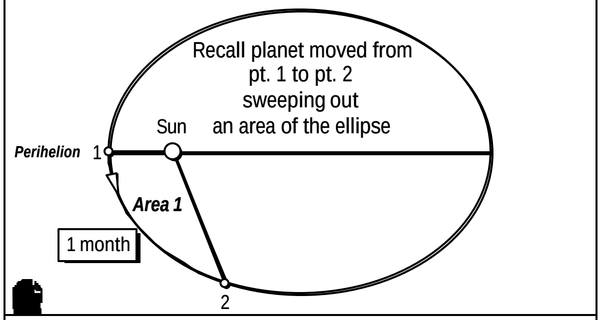
The Area Swept Out



Example: Law of Areas



Equal Areas in Equal Times



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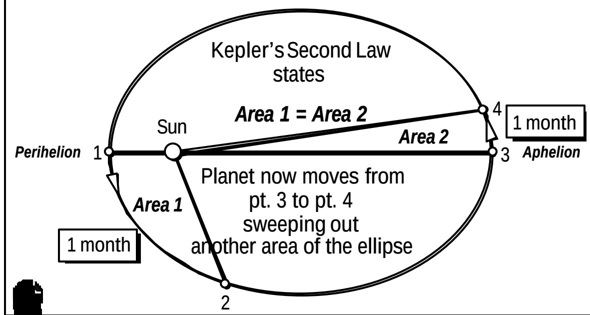
Equal Areas in Equal Times



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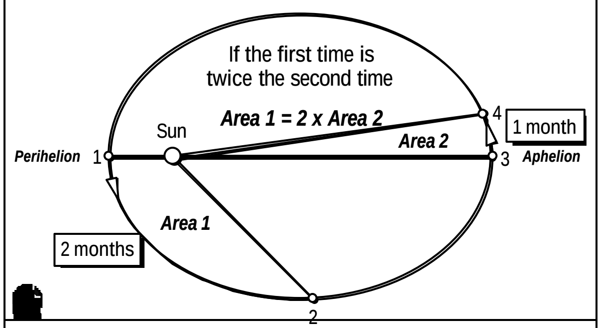
Areas Proportional to Times



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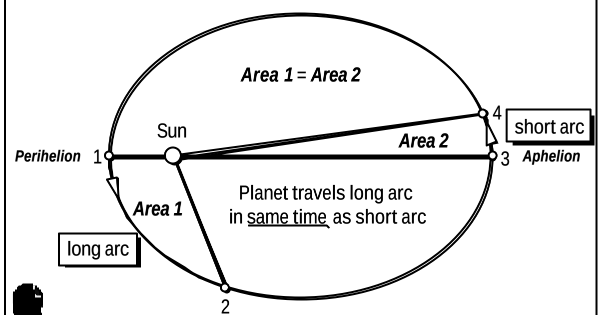
Significance of Law of Areas



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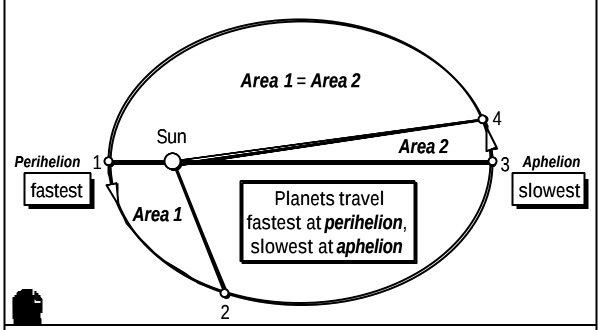
Conclusion



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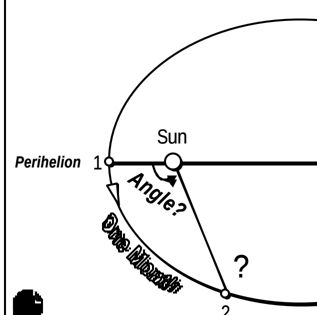
Why Second Law Uses Areas



Problem . . .

- Planet travels for a given time (say 1 month)
- Where is planet? (i.e., where is point 2 located?)
- Need to know ***angle***

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Need Angle at Sun

- Kepler unable to find angle & time relation

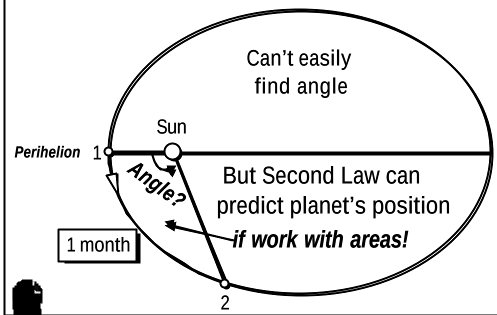
  Angle Û Time (?)

  ➢ ***No relation (formula) known to exist!***

- Only relation known between ***areas & time***

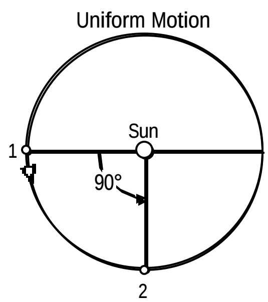
  Area µ Time !

Law Two "Most Important"



Find Planet — Circular Orbit

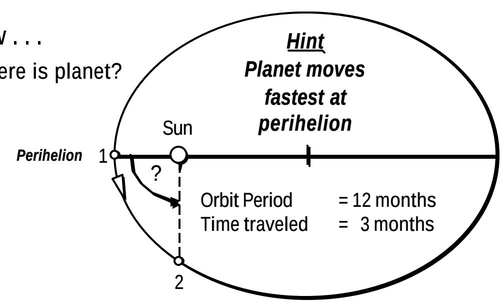
- Assume
  - Orbit Period = 12 months
  - Time traveled = 3 months
- Planet initially at pt. 1
- Where is planet now? (where is pt. 2?)
- Answer — 90° from pt. 1
  - 3 months/12 months = 1/4
  - So 1/4 of 360° = 90°



Find Planet — Elliptical Orbit

- Now . . .

  Where is planet?

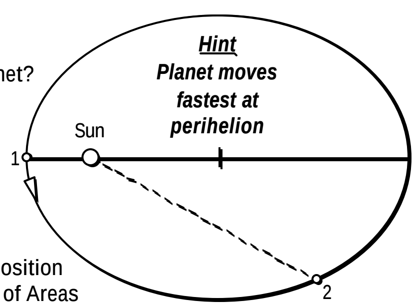


Find Planet — Elliptical Orbit

- Now . . .

  Where is planet?

- Answer —



But precise position requires Law of Areas

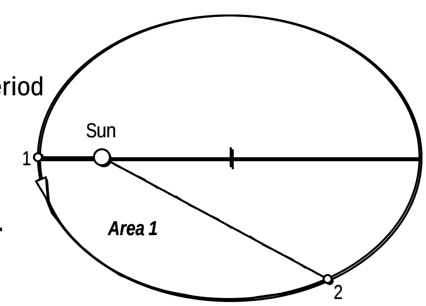
For Precise Position

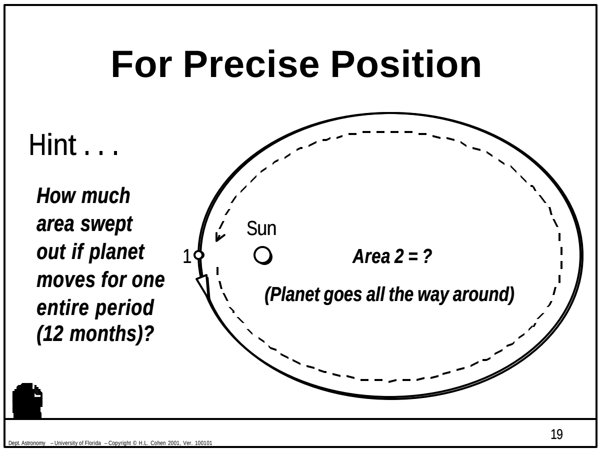
- Need ***areas***
- 1/4 orbit period used

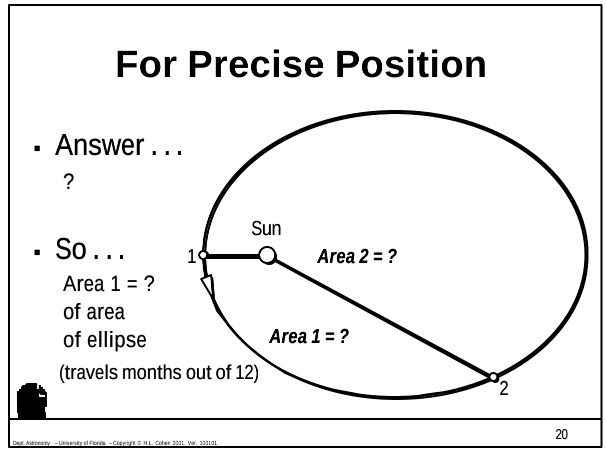
  (3 out of 12 months)

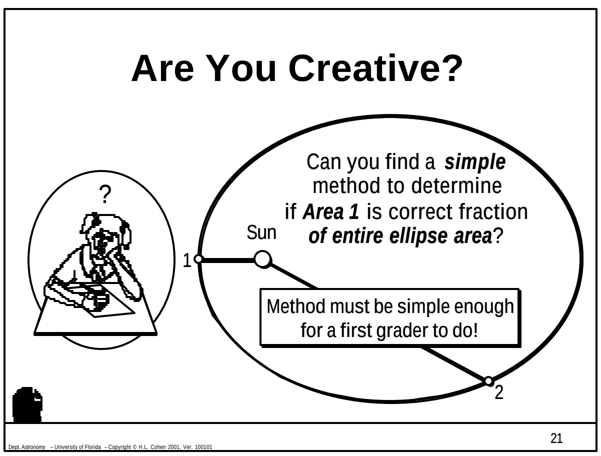
- So want . . . Area 1 = ?

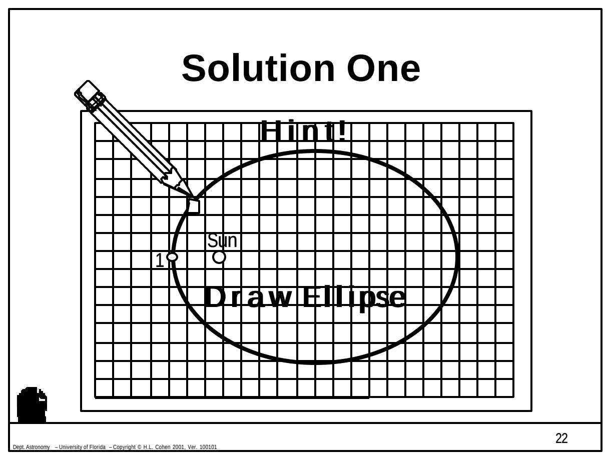
  (Hint follows)

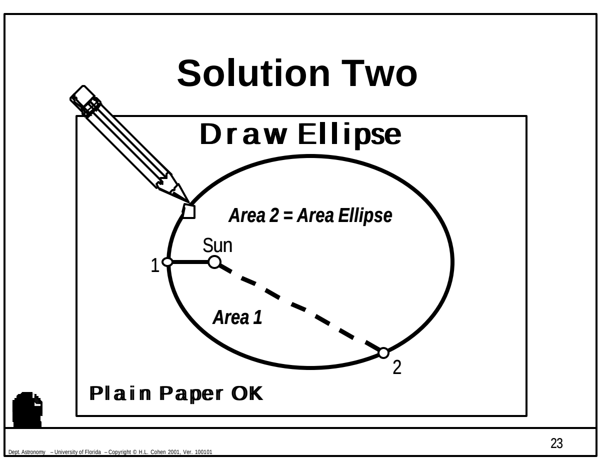


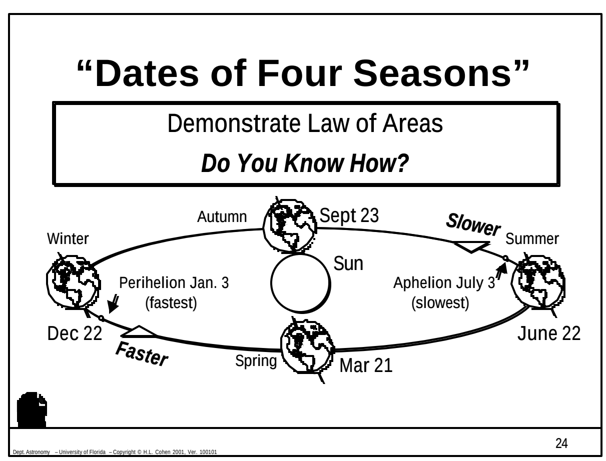

For Precise Position
Hint . . .
How much area swept out if planet moves for one entire period (12 months)?
Sun
1
Area 2 = ?
(Planet goes all the way around)
19


For Precise Position
Answer . . .
?
So . . .
Area 1 = ? of area of ellipse
(travels months out of 12)
Sun
1
2
Area 2 = ?
Area 1 = ?
20


Are You Creative?
?
Can you find a simple method to determine if Area 1 is correct fraction of entire ellipse area?
Sun
1
2
Method must be simple enough for a first grader to do!
21


Solution One
Hint!
Sun
1
Draw Ellipse
22


Solution Two
Draw Ellipse
Area 2 = Area Ellipse
Sun
1
Area 1
2
Plain Paper OK
23


"Dates of Four Seasons"
Demonstrate Law of Areas
Do You Know How?
Autumn
Sept 23
Slower
Winter
Summer
Sun
Perihelion Jan. 3 (fastest)
Aphelion July 3 (slowest)
Dec 22
June 22
Faster
Spring
Mar 21
24

Kepler's Third Law

Kepler's Third Law

- Alternate Name — ***Harmonic Law***
- Statement . . .

  "The square of the orbit period is proportional to the cube of the mean distance"
- Historical Note — publ. 10 yrs. after others
- Published 1619 in "***Harmony of the World"***

Summary

Let . . .

P = orbit period, (not ***perihelion***)

a = mean distance,

then P^2 µ a^3

Remember µ means ***proportional to***

The Solar System

(as known to Kepler)

Planet	Orbit Period		Mean Distance
Mercury	0.24 yr	Relation?	0.39 AU
Venus	0.62 yr		0.72 AU
Earth	1.00 yr		1.00 AU
Mars	1.88 yr		1.52 AU
Jupiter	11.9 yr		5.20 AU
Saturn	29.4 yr		9.56 AU

What is Relation?

Planet	Orbit Period		Mean Distance
Mercury	0.24 yr	Is it simply	0.39 AU
Venus	0.62 yr		0.72 AU
Earth	1.00 yr	***P*** µ ***a*** ?	1.00 AU
Mars	1.88 yr		1.52 AU
Jupiter	11.9 yr		5.20 AU
Saturn	29.4 yr		9.56 AU

The Solar System

Compare Earth to Jupiter

Planet	Orbit Period	Mean Distance
Mercury	0.24 yr	0.39 AU
Venus	0.62 yr	0.72 AU
Earth	1.00 yr	1.00 AU
Mars	1.88 yr	1.52 AU
Jupiter	11.9 yr	5.20 AU
Saturn	29.4 yr	9.56 AU

P about 12x larger

but ***a only*** about 5x larger!

What is Relation?

Planet	Orbit Period	Mean Distance
Mercury	0.24 yr	0.39 AU
Venus	0.62 yr	0.72 AU
Earth	1.00 yr	1.00 AU
Mars	1.88 yr	1.52 AU
Jupiter	11.9 yr	5.20 AU
Saturn	29.4 yr	9.56 AU

Does not work!

~~$P \propto a$?~~

What is Relation?

Planet	Orbit Period	Mean Distance
Mercury	0.24 yr	0.39 AU
Venus	0.62 yr	0.72 AU
Earth	1.00 yr	1.00 AU
Mars	1.88 yr	1.52 AU
Jupiter	11.9 yr	5.20 AU
Saturn	29.4 yr	9.56 AU

Kepler finds

$P^2 \propto a^3$

Kepler's Third Law

Compare Earth to Jupiter using $P^2 \propto a^3$

Planet	Orbit Period	Mean Distance
Mercury	0.24 yr	0.39 AU
Venus	0.62 yr	0.72 AU
Earth	1.00 yr	1.00 AU
Mars	1.88 yr	1.52 AU
Jupiter	11.9 yr	5.20 AU
Saturn	29.4 yr	9.56 AU

$\frac{11.9^2}{1.00^2}$ $\frac{5.20^3}{1.00^3}$

Kepler's Third Law

Results same!

Planet	Orbit Period	Mean Distance
Mercury	0.24 yr	0.39 AU
Venus	0.62 yr	0.72 AU
Earth	1.00 yr	1.00 AU
Mars	1.88 yr	1.52 AU
Jupiter	11.9 yr	5.20 AU
Saturn	29.4 yr	9.56 AU

$\frac{141}{1}$ $\frac{141}{1}$

(Results rounded to nearest integer)

Side Note

Need not write 1.0 in denominator if compare to Earth!

Planet	Orbit Period	Mean Distance
Mercury	0.24 yr	0.39 AU
Venus	0.62 yr	0.72 AU
Earth	1.00 yr	1.00 AU
Mars	1.88 yr	1.52 AU
Jupiter	11.9 yr	5.20 AU
Saturn	29.4 yr	9.56 AU

$\frac{11.9^2}{\text{~~1.00}^2\text{~~}}$ $\frac{5.20^3}{\text{~~1.00}^3\text{~~}}$

So write $P^2 = a^3$ if "***P***" in years, "***a***" in AU's

All Planets

Try it for yourself

Planet	P (years)	P^2	a (AU)	a^3
Mercury	0.24	0.058	0.39	0.058
Venus	0.62	0.378	0.72	0.378
Earth	1.00	1.00	1.00	1.00
Mars	1.88	3.53	1.52	3.51
Jupiter	11.9	141	5.20	141
Saturn	29.4	864	9.56	874
Uranus	83.8	7,020	19.2	7,080
Neptune	164	26,900	30.1	27,300
Pluto	248	61,500	39.4	61,200

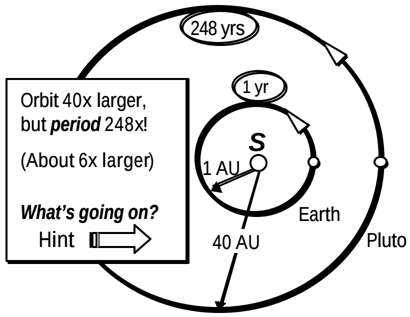
Nearly Same

Implications of Third Law

Draw

- Earth's orbit
- Pluto's orbit

Notice:
a = 40 AU
but
P = 248 yrs
(***not*** 40 yrs!)



Orbits not drawn to scale

Implications of Third Law

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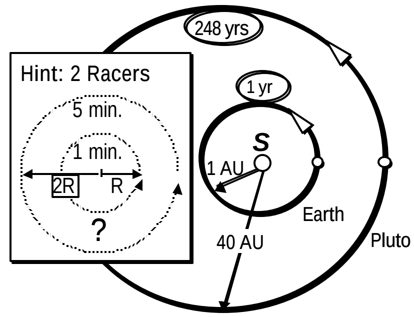

Orbits not drawn to scale

Implications of Third Law

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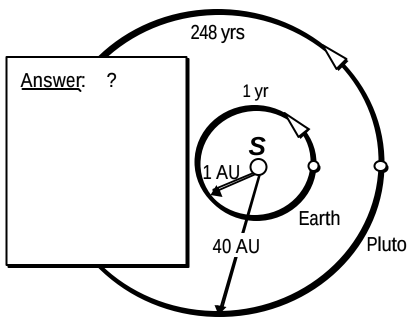


Orbits not drawn to scale

Conclusion

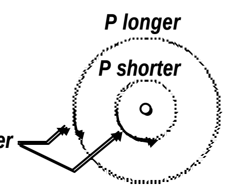
Kepler's Third (***Harmonic***) Law implies . . .

Two reasons outer planets take longer to orbit

1) Orbits ***larger***

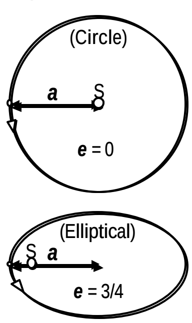
also

2) Average orbit speed ***slower***



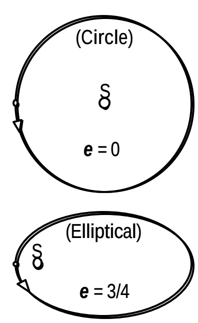
Try Answering This

- Two orbits
- Sizes identical
(“***a***” same)
- Shapes different
  - ***e*** = 0.00
  - ***e*** = 0.75
- ***Which planet takes longer to orbit Sun?***



Hint . . .

- Third (Harmonic) Law is
$P^2 = a^3$
- Answer ?
See if you can figure it out



Know

- How to use ***Third Law***: $P^2 = a^3$

  Remember — to write ***Third Law*** as above, must have
  - ***"P"*** in years
  - ***"a"*** in AU's
- Be able to find
  - ***"P"*** if given "***a***" . . . . . use as $P^2 = a^3$
  - ***"a"*** if given "***P***" . . . . . use as $a^3 = P^2$

  } Reverse

See ***Study Guide***: Last Two Slides in PowerPoint Handout
Example Problem 2 & Practice on Topic B Questions

Hints: Solving Third Law

- Find ***"P"*** : $P^2 = a^3$
  - Either $P = \sqrt{a^3}$ (Cube first)
  - or $P = \sqrt{a \times a \times a} = a\sqrt{a}$ (Sq. root first)
- Find ***"a"*** : $a^3 = P^2$
  - Either $a = \sqrt[3]{P^2}$ (Sq. first)
  - or $a = \sqrt[3]{P \times P} = \sqrt[3]{P} \times \sqrt[3]{P}$ (Cube root first)

Example: Solving Third Law

- Find ***"P"*** : $P^2 = a^3$ . . . if a = 25 AU
  - Either $P = \sqrt{25^3} = \sqrt{15{,}625} = 125$ yrs
  - or $P = \sqrt{25 \times 25 \times 25} = 25\sqrt{25} = 125$ yrs
- Find ***"a"*** : $a^3 = P^2$ . . . if P = 125 yrs
  - Either $a = \sqrt[3]{125^2} = \sqrt[3]{15{,}625} = 25$ AU
  - or $a = \sqrt[3]{125 \times 125} = \sqrt[3]{125} \times \sqrt[3]{125} = 25$ AU

