

Topic B

“The New Astronomy”


Part 4. Newton

(Web Version: 08-15-01)

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Issac Newton (1643–1727)

- English
 - Mathematician
 - Scientist
 - Head of Mint
- Born
 - 1 year after Galileo’s death
 - 13 years after Kepler’s death



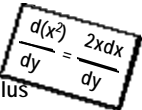

See Study Guide Fig. 3

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Issac Newton (1643–1727)

Contributions

- Invented differential calculus
- Formulated laws of mechanics & gravitation
(*Philosophiae Naturalis Principia Mathematica*, 1687)
- Theories of light & color (*Opticks*, 1704)
- Invented “Newtonian” reflecting telescope

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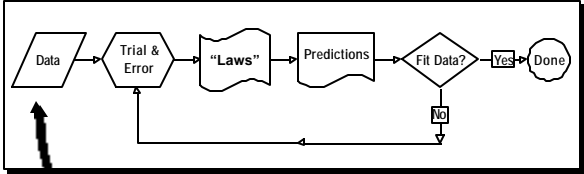
Issac Newton (1643–1727)

- Defensive of discoveries
- Sometimes reluctant to credit others
(Robert Hooke may have provided some of his ideas)
- Reluctant to publish
(Edmund Halley funded publication of *Principia*)
- Spent most of life on
Alchemy, theology & biblical chronology

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Kepler’s Approach

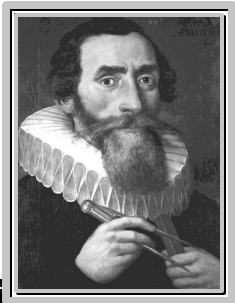
(Empirical Method)



Tycho’s Observations

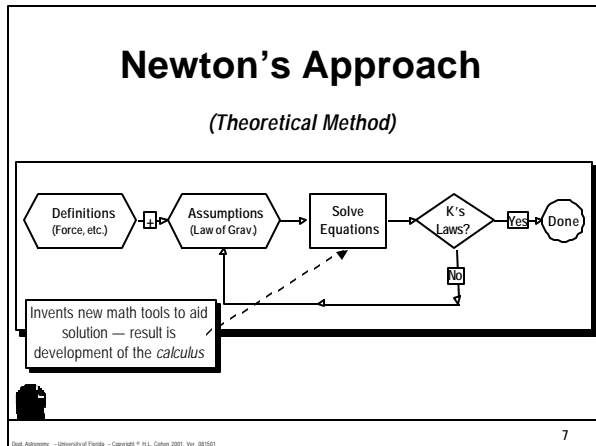
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Kepler’s Success



- ✓ Gradual adoption during 2nd half of 17th century
- ✓ Why?
 - Predictions much better
 - Tests/finds valid for other planets besides Mars
 - Newton provides “cause” (allows basis for mechanics)

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Newton's 2nd Law of Motion

Defines relation between *Force, Mass & Acceleration*

$$F = \text{mass} \times \text{acceleration}$$

- The *Effect* — Acceleration
- The *Cause* — Force (F)
- The *Mass* — Property of matter that resists acceleration
(Thus forces required to accelerate masses)

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Newton's Law of Gravitation

Postulates *a specific type of force* acting between masses

m — First Mass

M — Second Mass

d — Distance
(between centers of 2 masses)

F_{Gravity} — Gravitational Force

$$F_{\text{Gravity}} \propto \frac{m \times M}{d^2}$$

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Inverse Square Law

#1 m $\xrightarrow{F_{\text{Gravity}}}$ M
Distance: d

#2 m $\xrightarrow{4F_{\text{Gravity}}}$ M
Distance: $1/2d$

#3 m $\xrightarrow{2F_{\text{Gravity}}}$ $1/2 M$
Distance: $1/2d$

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- ### Results
- Newton derives *Kepler's Three Laws*
 - Derivations show each law . . .
More "general" or "elaborate" than Kepler realized
 - Briefly here is how each differs . . .
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- ### Law One: Newton's Version
- Possible orbits . . .
Any *conic section* if force varies inversely as distance squared
 - Forces need *not* be gravitational
 - *Conic sections* are . . .
 - circle
 - ellipse
 - parabola
 - hyperbola
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The Conic Sections

- Why so named?
- Answer . . .

Right circular *cone* cut by a *plane* forms these curves

- circle (c)
- ellipse (e)
- parabola (p)
- hyperbola (h)

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Examples . . .

- Some *comets* observed leaving solar system on *parabolic* or *hyperbolic* orbits
- Two *charged particles* (same sign) repel each other along *hyperbolic* orbits

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Law Two: Newton's Version

Law of Areas obeyed for any "central force"

(Central forces always point toward same point)

So, an "inverse square law" force unnecessary

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Law Three: Newton's Version

- For *Gravitational Force* $F \propto \frac{m \times M}{d^2}$
 m & M = two masses,
 d = distance
- Harmonic Law was $P^2 = a^3$

where P = orbit period & a = mean distance

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Law Three: Newton's Version

- For *Gravitational Force* $F \propto \frac{m \times M}{d^2}$
 m & M = two masses,
 d = distance
- Harmonic Law becomes $(M + m) P^2 = a^3$

New factor containing sum of the two masses

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Why Kepler "Had No Mass"

- For Sun and planet $(M + m) P^2 = a^3$
 - M = Sun's mass
 - m = planet's mass
- Assume m small compared to M
 - Result . . . can omit m if approximating
 - So (regardless of planet) $M P^2 = a^3$

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Why?

- Because
 - $M + m = \text{Big No.} + \text{Small No.}$
 - $= \text{Big No. (approximately)}$
- Example
 - $1.000 + 0.001 = 1.001$
 - Sun's Mass** = 1.000 (approximately!)
 - Planet's Mass**

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Example: Harmonic Law

Compare Earth to Jupiter using $(M+m)P^2 \propto a^3$

Planet	Mass*	Period	Mean Distance
Mercury		0.24 yr	0.39 AU
Venus		0.62 yr	0.72 AU
Earth	0.000003	1.00 yr	1.00 AU
Mars		1.88 yr	1.52 AU
Jupiter	0.000954	11.9 yr	5.20 AU
Saturn		29.4 yr	9.56 AU

*Mass compared to Sun's mass (i.e., mass Sun = 1 solar mass)

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Conclusion

- If Sun's mass (M) large compared to mass (m), can write
 - $(M+m)P^2 = a^3$
 - as $MP^2 = a^3$
 - And, if use $M = 1$ solar mass, $P^2 = a^3$
- Thus, Kepler doesn't know should include mass!

So, why is Harmonic Law useful?

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Usefulness of Harmonic Law

Compare objects orbiting different masses

Result — effect of mass shows up

Usefulness — *allows finding mass*

Examples?

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Example: Apply $M P^2 = a^3$

- System #1
 - $1 \times 1^2 = 1^3$
 - $1 \times 1 = 1$
 - $1 = 1$
- System #2
 - $4 \times P^2 = 1^3$
 - $4 \times P^2 = 1$
 - $P^2 = 1/4$

Same

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Why is P Smaller?

If star has *larger mass* . . .

- Gravitational force *higher* (Law of Gravitation, $F \propto Mm / d^2$)
- Planet *accelerates* (Newton's 2nd Law, $F = ma$)
- Orbit *speed higher*
- So *less time* to orbit

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Newton Provides "Cause"

Now we can see "cause and effect"

- The Cause . . . Force of Gravity (causes acceleration)
- The Effect . . . The Harmonic Law, $(M + m) P^2 = a^3$

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Earth-Moon System

- Moon: Earth's large satellite
- Orbit:
 - $a = 400,000$ km (approx.)
 - $P = 27$ days (approx.)
- Other planets have satellites too . . .

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Jupiter-Io System

- Io
 - One of Jupiter's large satellites
 - Discovered by Galileo (1610)
- Orbit
 - $a = 400,000$ km (approx.)
 - $P = 2$ days (approx.)

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Compare the Systems

Orbit sizes similar but orbit periods *very different* . . . Why?

Answer: ?

Result: ?

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Importance of Harmonic Law

$$(M + m) P^2 = a^3$$

- Used to find masses
 - Planets . . . if have at least one satellite
 - Stars if have companion (a "binary star")
 - All info about stellar masses from *Harmonic Law*
- *Harmonic Law* "priceless"

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Side Note

To apply Harmonic Law . . .
need at least one planetary satellite

➤ Which planets do not have any satellites?

• ?

• ?

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