

For this homework set, you'll need to do several simulations that may take your computer several anywhere from an hour to overnight. *Please be sure to start this soon enough that you'll be able to let your computer run overnight for at least four simulations* (one for each of your systems in #1d and #3e). And that's if you get everything right the first time! For each question, assume that all planets follow coplanar, edge-on orbits.

1. Use the systemic console¹ to analyze the observations of each of your planetary systems. For each of your planetary systems: (16 points)
 - a) Pretend that you did not know of the published orbital solution. Try to find an orbital solution of comparable (or better) quality than the published solution, but using the same (or fewer) number of planets. Report the orbital parameters of your solution.² Save this fit (you'll reuse it below, but also for questions #3 and #4).
 - b) Compare the parameters, chi-square, and rms deviation between the data and model based on your model to the published values and uncertainties. Note whether your model differs from the published solution by more than one would expect based on the published uncertainties.
 - c) Perform a bootstrap calculation to estimate the uncertainties for your model, using the "Bootstrap window". Note whether any of these differ substantially from the published uncertainties.
 - d) Perform n-body integrations to test for orbital stability, using the "stability window".³ Do your results suggest that this orbital solution would be unlikely to be stable for billions of years?

2. Based on Eqn. 26 of Cumming 2003 MNRAS 354, 1165 (you may assume $M/F \sim 10^6$) and a coplanar orbit, what is K_0 for each of your stars? How does this compare to the velocity amplitude due to each of the known planets? Consider the possibility that each of your stars has an additional planet following a circular orbit. How massive could such a planet be and still have a 50% chance of not being detected from existing radial velocity observations for each of the following hypothetical planets: (10 points)
 - a) an orbital period of 3 days,
 - b) an orbital period of 12 years,
 - c) in the "habitable zone"^{4*},
 - d) an orbital period half the orbital period of the least massive known planet,
 - e) between two planets (only applicable to multi-planet systems).

¹ You'll need to use either: 1) Local version of java program installed at /astro/depot/systemic/systemic, 2) [Downloadable java program](#), or 3) [Online java applet](#) (recently had problems accessing this version). If you'd like some help getting started, see tutorials [1](#), [2](#), and [3](#).

² You can list them or print out a screen shot with the relevant info.

³ I suggest you start with relatively short integrations ($\sim 10^{2-3}$ years) to help you rapidly eliminate solutions that are violently unstable. If you find a solution which appears stable over such timescales, then perform an integration of $\sim 10,000-100,000$ years (or however far your computer gets "overnight"). Use "save progress output to files". Change the base name so the integrations don't overwrite each other.

⁴ For simplicity, I'll suggest that you define the habitable zone to be star-planet separations between 0.9 and 1.2 AU for a solar mass star, and scale that distance with the square root of stellar luminosity.

3. For each of your planetary systems, evaluate the plausibility of there being an additional planet in each of the following orbits⁵: (24 points)

- i) an orbital period of ~12 years,
- ii) in the "habitable zone"⁶,
- iii) an orbital period half the orbital period of the least massive known planet,
- iv) between two planets (only applicable to multi-planet systems).

For each case: (16 points)

- a) Assuming the planet has a mass equal to the mass-detection threshold estimated in question #2, determine how much adding a planet of this mass and orbital period affects the chi-squared and rms deviation between the observations and the model (after allowing other orbital parameters to vary to improve the quality of the fit).
- b) Now allow the planet mass to vary to achieve an even better fit. How much did chi-squared and the rms deviation change (relative to original model, not part a)? Does the improved fit use a significantly larger or smaller planet mass? Save these fits so you can reuse them in parts c, e and f & question 4.
- c) Using the "F-test window" (and not "F-test significance window"), evaluate whether the improvement in the quality of the fit (relative to your best fit from question #1) is likely to be significant.
- d) Based on your analysis of the existing observational data, long-term orbital stability, and any other relevant physical constraints, how plausible is such a planet?

Now for the most plausible orbital solutions (i.e., at least one per system; more only if there are multiple viable solutions) from a-e: (8 points)

- e) Perform n-body integrations to test for orbital stability, using the "stability window"³. Do your results suggest that this orbital solution would be unlikely to be stable for billions of years?
- f) Use the bootstrap method to estimate uncertainties for each of the model parameters. Report the best-fit solution and uncertainties.

4. For each system, consider your best fit from question #1d and your best fit from question #3e. Use the output files ([BaseName]pl?.dat) from your long-term integrations of these systems: (24 points)

- a) Make plots of the semi-major axes (column 2), eccentricities (column 3), and longitude of pericenter (column 5) as a function of time (column 1).⁷
- b) Does the orbital evolution appear regular or likely to lead to instability?
- c) What is the range of eccentricities for each planet?
- d) If the evolution is regular, what is the timescale for eccentricity/pericenter evolution?⁸
- e) If the evolution is regular, plot the difference in the longitudes of pericenters for each pair of planets (within one system). Are any of these angles librating? If so, for which pairs of planets, and what is/are the amplitude(s) of libration?
- f) Based on the above results for your multiple planet system, how sensitive is the inferred secular evolution to the potential discovery of an additional planet in the system?

⁵ Start from your best-fit from problem one, add a planet, optimize the parameters of the new planet (holding the parameters for the previous planets fixed), and then optimize the parameters for all planets.

⁶ For simplicity, I'll suggest that you define the habitable zone to be star-planet separations between 0.9 and 1.2 AU for a solar mass star, and scale that distance with the square root of stellar luminosity.

⁷ Use your personal favorite program for making the plots in question #4.

⁸ If not, write the system name and not regular, so I know you didn't forget.