Protostars 2.

- Interior evolution:
  - Stellar Structure
  - Deuterium burning

Stahler & Palla: Chapter 11.2

Interior Evolution: Stellar Structure

[The general ideas presented here are important; you will not be expected to solve these equations in this class]

Protostars can be examined with the Stellar Structure Equations inside the accretion shock front. These conditions can then be matched with those in the infalling envelope.

The stellar structure equations for the dependent variables $r, P, T$ and $L_{\text{int}}$ are:

- [the interior mass $M_r$ acts as the spatial variable]

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4}$$

Diffusion equation

$$T^3 \frac{\partial T}{\partial M_r} = -\frac{3\kappa L_{\text{int}}}{256\pi^2 \sigma_B r^4}$$

Heat equation

$$\frac{\partial L_{\text{int}}}{\partial M_r} = \epsilon - T \frac{\partial s}{\partial t}$$

They must be supplemented by the equation of state and by knowledge of $\mu$, $\kappa$, $\epsilon$ and $s$ as a function of $\rho$ and $T$. 

The general ideas presented here are important; you will not be expected to solve these equations in this class.
Solution of the stellar structure equations requires specifications of four boundary conditions:

1. \( r(0) = 0 \)
2. \( L_{\text{int}}(0) = 0 \)

\( P(M_r) \) must equal the postshock value when \( M_r = M_\ast \). This is the ram pressure due to infalling matter \( (\rho u^2) \):

\[ \rho = \frac{\dot{M} r^{-3/2}}{4\pi \sqrt{2GM_\ast}} \]

\[ u = -V_{\text{ff}} = \left( \frac{2GM_\ast}{R_\ast} \right)^{1/2} \]

\[ \Rightarrow 3. \quad P(M_\ast) = \frac{\dot{M}}{4\pi} \left( \frac{2GM_\ast}{R_\ast^5} \right)^{1/2} \]

The fourth boundary condition concerns the surface value of the temperature and its relation to the luminosity. For a protostar:

\[ L_\ast = L_{\text{acc}} + L_{\text{post}} \]

\( L_{\text{post}} \) is the value of \( L_{\text{int}} \) obtained by integration (from \( M_r = 0 \) to \( M_\ast \)) of:

\[ \frac{\partial L_{\text{int}}}{\partial M_r} = \varepsilon - T \frac{\partial s}{\partial t} \]

and refers to the inner border of the postshock region. \( T_{\text{post}} \) is the corresponding temperature, as found by outward integration of the diffusion equation:

\[ T^3 \frac{\partial T}{\partial M_r} = -\frac{3kL_{\text{int}}}{256\pi^2 \sigma_B r^4} \]

How to relate \( T_{\text{post}} \) to \( L_{\text{post}} \) and \( L_{\text{acc}} \)?
Contributions to the luminosity of a low-mass protostar:

\[ L_{\text{post}} = 4\pi R^2 \sigma_B T_{\text{post}}^2 - 3L_{\text{acc}}/4 \]

\( L_{\text{post}} \) is the sum of inward and outward contributions. The outward luminosity \( (L_{\text{out}}) \), stemming from the deeper interior, is given by the blackbody formula. The hot gas immediately behind the shock emits in soft X-rays isotropically, whereas optical photons are stemming from the radiative precursor. The inward contribution to \( L_{\text{post}} \) includes \( L_{\text{acc}}/2 \) in X-rays and \( L_{\text{acc}}/4 \) in optical photons.

Mass-radius relation

The final ingredient needed to construct protostar models is the mass accretion rate, which enters the boundary conditions 3 and 4 and tells us how much to increase \( M \) from one time step to the next. Most studies have considered only constant rates, derived from the inside-out collapse model and assuming cloud temperatures of 10-20 K (\( \dot{M} \approx 10^{-8} - 10^{-6} M_\odot \text{ yr}^{-1} \)).

In practice, the four structure equations can be solved at any time by guessing central values for \( T \) and \( P \), using the boundary conditions 1 and 2, integrating outward (and keeping track of \( s \)). The guesses are altered until conditions 3 and 4 are met.
Onset of Convection

A fluid element, initially with the same density as its surroundings, moves a distance $\Delta r$ opposite to the gravitational acceleration $g$. To maintain pressure balance, the fluid element must expand.

If the drop in density is large enough for the element to continue to travel upward, the protostar is **convectively unstable**.

If it will sink back down (density still larger than surroundings) the protostar is **radiatively stable**. As long as $s(M_r)$ is an increasing function, the latter condition holds:

If the small displacement occurs so quickly that the element loses negligible heat through radiation, its specific entropy cannot change. A rising entropy profile in the external medium implies $(s_{\text{int}})_1 < (s_{\text{ext}})_1$. Internal and external pressures are equal. For ordinary gases, at fixed pressure, $(\partial p / \partial s)_p < 0 \Rightarrow (\rho_{\text{int}})_1 > (\rho_{\text{ext}})_1$.

A rising entropy profile implies radiative stability.

In our protostar, heat is transported outward by radiation rather than the convective motion of fluids elements. But this situation cannot last forever...

The ratio $M_r / R_r$ rises and the interior temperature also climbs:

$$T \approx \frac{\mu \, GM}{3 R R}$$

Nuclear reactions eventually begin near the center, where the temperature peaks. Their effect is to increase the central entropy, until the profile overturns ($\varepsilon$ grows large near $M_r = 0$; since $L_{\text{int}}$ is kept relatively small by the high optical depth, the specific entropy increases with time):

$$\frac{\partial L_{\text{int}}}{\partial M_r} = \varepsilon - T \frac{\partial s}{\partial t}$$

The important physical consequence is that the protostar becomes convectively unstable.
When the central temperature reaches $\sim 10^6$ K, deuterium fusion becomes appreciable. The first nuclear fuel to ignite is a small admixture of $^2$H, which is destroyed by fusion with protons ($\Delta E_D=5.5$ MeV):

$$^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma$$

- Convection begins in protostars because deuterium fusion produces too much luminosity to be transported radiatively through the highly opaque interior.
- Discrete “cells” of fluid rise toward the surface.
- Since $\partial s/\partial M_r < 0$, the rising fluid is now both underdense and hot relative to its surroundings and it transfers the excess heat to this medium.
- The cooler and denser cell sinks back down and the cycle repeats.

From a computational perspective, the onset of convection modifies the thermal stellar structure equations (e.g. the diffusion equation is no longer valid). Traditionally, the semi-empirical mixing-length theory is used.

Every cell at a given radius $r$ expels its heat after traversing the same distance. The “mixing length” is usually taken to be some factor of order unity times the local pressure scale height (i.e. the radial distance over which $P(r)$ would drop by $e^{-1}$). The transport of heat through interior motion is highly efficient.

Convective instability is a local phenomenon and the computational procedure must account for this fact by testing for stability at each $M_r$-value. A transition from convection to radiative diffusion always occurs at least once before the surface is reached (the falling density eventually renders convection inefficient).

The outermost layers of any star are radiatively stable.

In the early phases of D-burning, most of the protostar is convective, which allows mixing of fresh D accreted to the surface down to the core where it undergoes fusion.
The entropy increase and swelling of the protostar stem from two distinct sources:

a. back-heating from the accretion shock;

b. internal heating from deuterium burning.

Deuterium shell burning

(a) central burning  (b) radiative barrier

(c) depleted interior  (d) shell burning
More massive protostars

After a transient swelling ~4-5 Msun, the protostar contracts, the central temperature rises until H fusion can begin ~$10^7$K.