Recap: The Jeans Mass and the free-fall time

A cloud that is supported by thermal pressure will collapse if its mass exceeds the Jeans mass, which is \( \propto T^{3/2} \rho^{-1/2} \):

\[
M_C > M_J \equiv \left( \frac{5kT}{G\mu m_H} \right)^{3/2} \left( \frac{3}{4\pi \rho_C} \right)^{1/2}
\]

The free-fall time is of order \((G\rho)^{-1/2}\):

\[
t_{ff} \equiv \left( \frac{3\pi}{32G\rho_C} \right)^{1/2}
\]
Isothermal Collapse

\[ M_J \sim 10^5 \frac{T^{3/2}}{\mu^2 n^{1/2}} M_\odot \]

If the core temperature remains constant during the initial stages of the collapse, the Jeans mass will decrease as density increases.

Let’s look at the cooling process:

\[
\text{Cooling time} = \frac{\text{thermal energy}}{\text{rate of loss of thermal energy}}
\]

\[
= \frac{u(Jm^{-3})}{\Lambda(Jm^{-3}s^{-1})}
\]

\[
= \frac{(3/2)nkT}{\Lambda}
\]

Recap: Heating and Cooling of Gas in Molecular Clouds

Main Heating Process: Cosmic Rays on H₂:

\[ p^+ + H_2 \rightarrow H_2^+ + e^- + p^+ \]

Cosmic rays mostly consist of relativistic protons, with an admixture of heavy elements and electrons. All the particles are electrically charged and thus subject to magnetic deflection.

Cosmic rays with energies up to \(10^9\) MeV are produced by particle acceleration within the magnetized shocks created by supernova remnants. More energetic particles are probably extragalactic in origin.
In a molecular cloud, a gyrating cosmic ray proton interacts with ambient nuclei and electrons through both the Coulomb and nuclear forces. The nuclear excitations \((E_p \geq 1 \text{ GeV})\) principally decay through emission of \(\gamma\) rays.

The proton scatters inelastically with \(H_2\), mainly ionizing it:

\[
p^+ + H_2 \rightarrow H_2^+ + e^- + p^+
\]

It is this secondary electron that provides heat through its subsequent interactions with ambient \(H_2\). The heat deposition in the cloud per unit volume (heating rate):

\[
\Gamma_{CR}(H_2) = \zeta(H_2)n_{H_2}\Delta E(H_2)
\]

Ionization rate (probability per unit time) = \(1-10 \times 10^{-17} \text{ s}^{-1}\)

The most important means for the electron to provide heating is dissociation:

\[
e^- + H_2 \rightarrow H + H + e^-
\]

The energy of the incoming electron beyond that required to dissociate \(H_2\) goes into motions of the two hydrogen atoms. Collisions quickly disperse this energy throughout the gas. In total, a 10 MeV proton provides:

\[
\Delta E(H_2) \approx 7 \text{ eV}
\]

\[
\Gamma_{CR}(H_2) \approx 3.4 \times 10^{-25} \left( \frac{\zeta(H_2)}{3 \times 10^{-17} \text{ s}^{-1}} \right) \left( \frac{n(H_2)}{10^{10} \text{ m}^{-3}} \right) \text{ J s}^{-1} \text{ m}^{-3}
\]
What is the dominant cooling agent in molecular clouds?

\( \Lambda_{\text{CO}} \) depends on the number density of CO molecules in the cloud, on the energy of the transition and on the optical depth of the emitted lines.

**CO cooling**

A simple case: the two-level system

\[
\begin{align*}
A_{ul} &= \frac{2h\nu_{0}^{3}}{c^{2}}B_{ul} \\
g_{l}B_{lu} &= g_{u}B_{ul} \\
\gamma_{lu} &= \frac{g_{u}}{g_{l}}\exp\left(-\frac{\Delta E}{k_{B}T_{\text{kin}}}\right) \\
\gamma_{ul} &= \frac{g_{u}}{g_{l}}\exp\left(-\frac{\Delta E}{k_{B}T_{\text{ex}}}\right)
\end{align*}
\]

The level populations of the upper and lower states \( (n_{u} \) and \( n_{l} \)) are given by the Boltzmann relation:

\[
\frac{n_{u}}{n_{l}} = g_{u} \exp\left(-\frac{\Delta E}{k_{B}T_{\text{ex}}}\right)
\]
**CO cooling**

If CO is excited by collisions with H$_2$ at the rate $n_\text{H}_2\gamma_{ul}$ per molecule, what is $\Lambda_{ul}$, the volumetric rate of cooling?

1. **Sub-critical (low-density) regime**: if $n_\text{H}_2 << n_{\text{crit}} \equiv A_{ul}/\gamma_{ul}$, each upward collisional excitation is followed promptly by a downward radiative transition. The cooling rate by H$_2$ impact is

$$\Lambda_{ul}(n_\text{H}_2 << n_{\text{crit}}) = n_in_\text{H}_2\gamma_{ul}\Delta E_{ul}$$

$$= \frac{g_u}{g_l} n_in_\text{H}_2\gamma_{ul}\Delta E_{ul} \exp\left(-\frac{\Delta E_{ul}}{k_BT_{\text{kin}}}\right)$$

\[x(\text{CO}) \equiv n(\text{CO})/n_\text{H}_2 \equiv 10^{-4}\]

$$n_l = x(\text{CO})n_\text{H}_2 \rightarrow \Lambda_{ul}(n_\text{H}_2 << n_{\text{crit}}) \propto n_\text{H}_2^2$$

2. **LTE (high density) regime**: if $n_\text{H}_2 >> n_{\text{crit}} \equiv A_{ul}/\gamma_{ul}$, each upward collisional excitation is followed by collisional deexcitation rather than by radiative decay. The two-level populations are in LTE and do not depend on $n_\text{H}_2$. The cooling rate is the product of $n_u$ and $A_{ul}\Delta E_{ul}$, the energy loss rate from the upper level:

$$\Lambda_{ul}(n_\text{H}_2 >> n_{\text{crit}}) = n_uA_{ul}\Delta E_{ul}$$

$$= \frac{g_u}{g_l} n_uA_{ul}\Delta E_{ul} \exp\left(-\frac{\Delta E_{ul}}{k_BT_{\text{kin}}}\right)$$

$$n_l = x(\text{CO})n_\text{H}_2 \rightarrow \Lambda_{ul}(n_\text{H}_2 >> n_{\text{crit}}) \propto n_\text{H}_2$$
CO cooling

For CO(1-0) transitions (see http://www.strw.leidenuniv.nl/~moldata/datafiles/co.dat):

\[ A_{10} = 7.2 \times 10^{-8} \text{ s}^{-1} \]
\[ \gamma_{01} = 3.3 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1} \]
\[ \nu_{10} = 115.27 \text{ GHz} \]

For a typical molecular cloud core, with \( n_{H_2} \sim 10^4 \text{ cm}^{-3} (=10^{10} \text{ m}^{-3}) > n_{\text{crit}} (=2.2 \times 10^3 \text{ cm}^{-3}) \) and \( T_{\text{kin}} = 10 \text{ K} \), we can use the LTE-regime formula, obtaining:

\[ \Lambda_{ul} = 9.5 \times 10^{-23} \text{ erg cm}^{-3} \text{ s}^{-1} \]

Assuming that the cooling is only due to CO (at \( n_{H_2}=10^{10} \text{ m}^{-3}, T = 10\text{K} \)), \( \Lambda_{\text{CO}} \sim 10^{-22} \text{ erg cm}^{-3} \text{ s}^{-1} \), and the cooling time is:

\[ t_{C} = \frac{(3/2)nkT}{\Lambda_{\text{CO}}} = 7 \times 10^3 \left( \frac{n}{2 \times 10^{10} \text{ m}^{-3}} \right) \text{ yr} \]

If we compare this to the free-fall time at the same density:

\[ t_{\text{ff}} = \left( \frac{3\pi}{32G\mu m_H n} \right)^{1/2} = 3 \times 10^5 \left( \frac{n^4}{2 \times 10^{10} \text{ m}^{-3}} \right)^{-1/2} \text{ yr} \]

Hence, \( t_{C} < t_{\text{ff}} \Rightarrow \text{the collapse is initially isothermal} \). As the core collapses and the density increases, the Jeans mass will decrease, allowing smaller sub-units to become unstable against collapse.
Fragmentation

Schematic

Fragmentation

Observational View

IRAS 19410+2336 - Beuther & Schilke, 2004, Science, 303, 1167
Fragmentation

The collapse of molecular clouds into stars takes place in stages:

• the large cloud begins to collapse when it becomes dense enough

• localized regions within the cloud also collapse independently of the larger cloud → fragmentation

When a fragment can collapse independently without further disruption, it will go on to form the protostar.

What stops the fragmentation?

During collapse, the cloud remains isothermal as long as the gravitational potential energy released during collapse is efficiently radiated away, i.e. the cloud must remain optically thin!

If the cloud becomes optically thick, it can no longer cool effectively. The Jeans mass increases as $T$ increases.

$$M_J \sim 10^5 \frac{T^{3/2}}{\mu n^{1/2}} M_\odot$$

Does the optical depth increase?
For thermal radiation, the opacity is dominated by dust: [but for CO line cooling it is dominated by line opacity]

\[ \Delta \tau_v = \rho \kappa_v \Delta s \Rightarrow \tau \propto nR \]

From the Jeans length:

\[ R_J \approx \frac{1}{\mu m_H} \left( \frac{15kT}{4\pi G n} \right)^{1/2} \]

\[ \sim \frac{10^4}{\mu} \left( \frac{T}{n} \right)^{1/2} \text{ pc} \]

Thus, \( \tau \propto n^{1/2} \) for constant \( T \) \( \Rightarrow \) \( \tau \) increases as \( n \) increases.

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**What is the mass of the smallest fragment?**

Once a fragment becomes opaque to its own radiation, it will radiate almost as a blackbody. To find the mass of the smallest fragment, consider that the rate of radiation loss \( \sim \) the rate of gain in gravitational energy:

\[ 4\pi R^2 \sigma T^4 \sim \frac{GM^2}{R} \frac{1}{t_{ff}} \]

Using the expression for \( t_{ff} \), \( R^3 = M/(4/3 \pi \rho) \) and the Jeans mass expression to eliminate \( \rho \):

\[ M \sim 0.007 \left( \frac{T^{1/4}}{\mu^{9/4}} \right) M_\odot \]

\( \Rightarrow \) If \( T = 10 \text{ K} \) and \( \mu = 2.4 \), \( M \sim 0.002 M_\odot \)
Summary

- Initial collapse is isothermal \((t_C < t_{ff})\)
- As Jeans mass decreases, smaller fragments can begin to collapse independently of the larger structure
- Smallest fragments could explain the full range of stellar masses (as we will see when we will talk about the IMF).