Interstellar Dust: Extinction and Thermal emission

The brightness of a star near any wavelength $\lambda$ is measured either by its apparent and absolute magnitudes: $m_\lambda$ and $M_\lambda$ respectively:

$$
\begin{align*}
    m_\lambda &= -2.5 \log F_\lambda(r) + m_\lambda^o \\
    M_\lambda &= -2.5 \log F_\lambda(10\,pc) + m_\lambda^o \\
    m_\lambda &= M_\lambda + 5 \log \left( \frac{r}{10\,pc} \right)
\end{align*}
$$

If dust is present along the line of sight:

$$
    m_\lambda = M_\lambda + 5 \log \left( \frac{r}{10\,pc} \right) + A_\lambda
$$

Extinction at $\lambda$

Consider two different wavelengths, $\lambda_1$ and $\lambda_2$:

$$
    (m_{\lambda_1} - m_{\lambda_2}) = (M_{\lambda_1} - M_{\lambda_2}) + (A_{\lambda_1} - A_{\lambda_2})
$$

Observed color index, $C_{12}$

Intrinsic color index, $C^0_{12}$

Color excess, $E_{12} = C_{12} - C^0_{12}$
The Interstellar Extinction Curve

Both extinction and the color excess are proportional to the column density of dust grains along the l.o.s. If we consider another wavelength $\lambda_3$, the ratios $A_{\lambda}/E_{12}$ and $E_{32}/E_{12}$ depend only on intrinsic grain properties. Let the third wavelength to have an arbitrary value:

$$\frac{E_{\lambda-V}}{E_{B-V}} = \frac{A_{\lambda}}{A_V} = \frac{A_{\lambda}}{E_{B-V}}$$

$$R = 3.1$$

(in the diffuse interstellar medium; but can be ~5 in dense clouds because grains are larger)

![Interstellar Extinction Curve Diagram]

Transfer of Radiation: Intensity, Flux, Energy Density

- **Specific intensity**, $I_\nu$: $\Delta E = I_\nu \Delta A_\nu \Delta t \Delta \Omega$

- **Specific flux** $F_\nu$, or flux density, the monochromatic energy per unit area per unit time passing through a surface of fixed orientation $z$ ($W \text{ m}^{-2} \text{ Hz}^{-1}$ or erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$ or Jansky = $10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$):

$$F_\nu = \int I_\nu \mu d\Omega$$

- **Total energy density** $u_\nu$, per unit frequency at a fixed location ($J \text{ m}^{-3} \text{ Hz}^{-1}$ or erg cm$^{-3}$ Hz$^{-1}$):

$$u_\nu = \frac{1}{c} \int I_\nu d\Omega$$

- **Mean intensity** $J_\nu$, i.e. the average of $I_\nu$ over all directions:

$$J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega = \frac{c}{4\pi} u_\nu$$
Assume that the radiation field travels along a small distance $\Delta s$. $I_\nu$ can be absorbed (radiative energy is transformed to internal motion of the grain lattice) and scattered (a photon with the same frequency is reemitted in a different direction):

$$\Delta I_{\nu 1} = -\rho \kappa_\nu I_\nu \Delta s$$

$\kappa_\nu$ is the opacity (cm$^2$ g$^{-1}$), a quantity that depends on the incident frequency $\nu$, the relative number of grains and their intrinsic physical properties.

$1/\rho \kappa_\nu =$ photon mean free path

$\rho \kappa_\nu =$ absorption coefficient (cm$^{-1}$)

The optical depth is defined by:

$$\Delta \tau_\nu = \rho \kappa_\nu \Delta s$$

$I_\nu$ can also increase due to the thermal emission from the dust:

$$\Delta I_{\nu 2} = +j_\nu \Delta s$$

$j_\nu$ is the emissivity (W m$^{-3}$ sr$^{-1}$ Hz$^{-1}$) (or erg s$^{-1}$ cm$^{-3}$ sr$^{-1}$ Hz$^{-1}$), such that $j_\nu \Delta \nu \Delta \Omega$ is the energy per unit volume per unit time emitted into the direction $n$. 
The equation of transfer

\[ \Delta I_v = \Delta I_{v1} + \Delta I_{v2} = -\rho \kappa_v I_v \Delta s + j_v \Delta s \]

\[ \frac{dI_v}{ds} = -\rho \kappa_v I_v + j_v \]

Source Function

\[ S_v = \frac{j_v}{\rho \kappa_v} \]
Optical depth ($\tau_\lambda$) and extinction ($A_\lambda$)

Let's use the equation of transfer to obtain the specific intensity at a point $P$ located at a distance $r$ from the center of a star. Assume that $j_\lambda = 0$. Integrating the equation along any ray from the stellar surface to $P$, we obtain:

$$I_\lambda(r) = I_\lambda(R_*) \exp(-\Delta \tau_\lambda)$$

Let's now determine the Flux density ($\mu = 1$, and $\Delta \Omega = \pi R_*^2 / r^2$):

$$F_\lambda = \int I_\lambda \mu d\Omega$$

$$F_\lambda(r) = \pi I_\lambda(R_*) \left( \frac{R_*}{r} \right)^2 \exp(-\Delta \tau_\lambda)$$

Assume now that the same star is located at $r_0$ from $P$, with no intervening extinction:

$$F_\lambda^*(r_0) = \pi I_\lambda(R_*) \left( \frac{R_*}{r_0} \right)^2$$

Dividing (a) by (b) and taking the log:

$$-2.5 \log F_\lambda(r) = -2.5 \log F_\lambda^*(r_0) + 5 \log \left( \frac{r}{r_0} \right) + 2.5 \log(e) \Delta \tau_\lambda$$

$$m_\lambda = M_\lambda + 5 \log \left( \frac{r}{10 \, pc} \right) + A_\lambda \Rightarrow A_\lambda = 2.5 \log(e) \Delta \tau_\lambda$$

$$A_\lambda = 1.086 \Delta \tau_\lambda$$
Blackbody Radiation

A black body is a theoretical object that absorbs 100% of the radiation that hits it. Therefore it reflects no radiation and appears perfectly black.

At a particular temperature the black body would emit the maximum amount of energy possible for that temperature. This value is known as the black body radiation. It also emits a definite amount of energy at each wavelength for a particular temperature, so standard black body radiation curves can be drawn for each temperature, showing the energy radiated at each wavelength.

\[
\begin{align*}
B_{\nu}(T) &= \frac{2h\nu^3 / c^2}{\exp(h\nu / k_B T) - 1} \\
B_{\lambda}(T) &= \frac{2hc^2 / \lambda^5}{\exp(hc / \lambda k_B T) - 1}
\end{align*}
\]

Blackbody radiation is radiation in thermal equilibrium and it is isotropic, i.e. the specific intensity \( I_\nu \) is independent of direction:

\[
I_\nu = \frac{1}{c} \int I_\nu d\Omega \Rightarrow I_\nu = cu_\nu / 4\pi \equiv B_\nu
\]
Blackbody Radiation

Stars are so optically thick at all frequencies that its matter and radiation are very nearly in thermal equilibrium. 

\[ I_\lambda (r_s) = B_\lambda (T_{\text{eff}}) \]

\[ F_\lambda = \int I_\lambda d\Omega \]

\[ d\Omega = 2\pi d\mu \]

\[ F_\lambda (R_s) = \pi B_\lambda (T_{\text{eff}}) \]

Wien's displacement law:

\[ \frac{v_{\text{max}}}{T} = \frac{2.82 k_B}{h} = 5.88 \times 10^{10} \text{Hz K}^{-1} \]

\[ \lambda_{\text{max}} T = 0.29 \text{ cm K} \]

Much of a person's energy is radiated away in the form of infrared light.
Blackbody Radiation

\[ F\lambda(R_s) = \pi B\lambda(T_{eff}) \]

\[ B\lambda(T) = \frac{2hc^2}{\lambda^5} \exp\left(\frac{hc}{\lambda k_B T}-1\right) \]

Integrating \( F\lambda(R_s) \) over all wavelengths, we obtain the bolometric flux \( F_{bol} \):

\[ F_{bol} = \frac{2\pi k_B^4 T_{eff}^4}{c^2 h^3} \int_0^\infty \frac{y^3 dy}{e^y - 1} \]

\[ \pi^4/15 \]

\[ F_{bol} = \frac{2\pi^5 k_B^4}{15c^2 h^3} T_{eff}^4 = \sigma_B T_{eff}^4 \]

\[ L_{bol} = 4\pi R_s^2 \sigma_B T_{eff}^4 \]

\[ \sigma_B = 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} \]

Stefan-Boltzmann constant

Interstellar Dust: Properties of the Grains

1. Efficiency of extinction

\[ \frac{dI_\nu}{ds} = -\rho \kappa_\nu I_\nu + j_\nu \]

The opacity \( \kappa_\nu \) represent the total extinction cross section per mass of interstellar material. To explore the contribution of each grain:

\[ \rho \kappa_\nu = n_d \sigma_d Q_\nu \]

\( n_d \equiv \) number of dust grains per unit volume (number density)

\( \sigma_d \equiv \) geometrical cross section of a typical grain

\( Q_\nu \equiv \) extinction efficiency factor

\[ \Delta \tau_\nu = \rho \kappa_\nu \Delta s \]

\[ A_\nu = 1.086 \Delta \tau_\nu \]

\[ Q_\nu \propto A_\nu / N_d \]
At far-infrared and millimeter wavelengths, the ISM is generally transparent, so one needs to observe the emission from heated dust clouds to determine $A_\lambda$ and $Q_\lambda$. From the equation of radiative transfer, ignoring the absorption term and assuming optically thin conditions:

$$I_\lambda = B_\lambda(T_d) \Delta \tau_\lambda$$
$$F_\lambda = B_\lambda(T_d) \Delta \Omega \Delta \tau_\lambda$$

Knowledge of $A_\nu$ and $T_d$ can give information on the wavelength dependence of $Q_\lambda$, but the determination of both $A_\nu$ and $T_d$ is usually problematic. It is conventional to write $Q_\lambda \propto \lambda^{-\beta}$, with $\beta \approx 1-2$ between $30 \mu m \leq \lambda \leq 1 mm$. $\beta$ is smaller in the densest clouds and circumstellar disks, but closer to 2 in more diffuse environments.

In general, the opacity and efficiency factor of a grain does not depend on $\lambda$ once the geometric size becomes larger than $\lambda$. Hence, centimeter-size grains have a small exponent $\beta$ in the millimeter.

### 2. Size distribution

$$dn_d = C n_H a^{-3.5} da, \ a_{\text{min}} < a < a_{\text{max}}$$

$$a_{\text{min}} = 50 \mu m, a_{\text{max}} = 0.25 \mu m$$

$$C = 10^{-25.13}(10^{-25.11}) \ cm^{2.5} \ for \ graphite \ (silicate)$$

Although most of the mass is in $0.1 \mu m$ grains, the surface area is mainly in small particles.

Useful parameter: $\Sigma_d$, the total geometric cross section of grains per hydrogen atom:

$$\Sigma_d = \frac{n_d \sigma_d}{n_H}$$

Mathis, Rumpl & Nordsieck 1977
Weingartner & Draine 2001
3. The dust-to-gas mass ratio

\[ \Sigma_d = \frac{n_d \sigma_d}{n_H} = \frac{N_d \sigma_d}{N_H} \]

\[ N_d \equiv n_d L; \quad N_H \equiv n_H L \]

\[ \rho \kappa_\lambda = n_d \sigma_d Q_\lambda \quad \Rightarrow \quad \Delta \tau_\lambda = N_d \sigma_d Q_\lambda \]

\[ \Sigma_d = \frac{\Delta \tau_\lambda}{Q_\lambda N_H} = \frac{A_\lambda}{1.086 Q_\lambda N_H} \]

When \( \lambda = V \), \( Q_\lambda = 1 \) (van der Hulst 1981; Tielens 2005):

\[ \Sigma_d = \frac{A_V}{1.086 N_H} = \frac{A_V / E_{B-V}}{1.086 N_H / E_{B-V}} = 2.9 \frac{E_{B-V}}{N_H} \]

From Bohlin, Savage & Drake (1978), \( E_{B-V}/N_H = 1.7 \times 10^{-22} \) mag cm\(^2\).

\[ \Sigma_d \approx 5 \times 10^{-22} \text{ cm}^2 \]

The mass fraction of the interstellar medium contained in grains, \( f_d \):

\[ f_d = \frac{m_d n_d}{m_{\text{gas}} n_{\text{gas}}} = \frac{4 \rho_d a_d \Sigma_d}{3 \mu m_H} \equiv 0.01 \]

\( f_d = Z \), the metallicity of the gas \( \Rightarrow \) a large portion of heavy elements must be locked up in solid form!
Observations at non-visible wavelengths reveal the shape of the Galaxy

At visible wavelengths ($\lambda$), light suffers so much interstellar extinction that the galactic nucleus is totally obscured from view. But the amount of interstellar extinction is roughly inversely proportional to $\lambda$.

As a result, we can see farther into the plane of the Milky Way at infrared $\lambda$ than at visible $\lambda$, and radio waves can traverse the Galaxy freely.

Starlight warms the dust grains to $T \sim 10$ to $90$ K. Thus, from Wien’s Law:

$$\lambda_{\text{max}} = \frac{0.0029 \text{ K m}}{T}$$

the dust emits predominantly at $\lambda \sim 30$ to $300$ $\mu$m. These are far-infrared (FIR) $\lambda$. At these $\lambda$, interstellar dust radiates more strongly than stars, so a FIR view of the sky is principally a view of where the dust is.