The existence of dense molecular clouds was one of the first clues to understand the formation of stars

• First identified as dark nebulae by William and Caroline Herschel (Herschel, 1785), but it was not until photographic observations of Barnard (1919) and Wolf (1923) were these objects established as discrete, optically opaque interstellar clouds.

• In 1950, the discovery of the HI line (21 cm emission) led to a correlation between dust absorption and HI emission (Lilley, 1955). BUT, observations of HI towards the centers of dark nebulae detected either very weak HI emission or found it to be totally absent.

• This led Bok et al. (1955) to suggest that if gas existed within these nebulae it had to be molecular in form.

• Cold molecular component of the interstellar medium (ISM) was discovered in the 1970 (e.g., Wilson et al. 1970) primarily through CO observations.

• Quickly realized that dark clouds were molecular clouds consisting almost entirely of molecular hydrogen mixed with small amounts of interstellar dust and trace amounts of more complex molecular species.

• Molecular clouds are the sites of all star formation in the Milky Way
Carbon Monoxide (CO) rotation

Kinetic energy of a rotating dumbbell:

\[ E_{\text{rot}} = \frac{J^2}{2I} \]

The quantum mechanical analog of \( E_{\text{rot}} \) is:

\[ E_{\text{rot}} = \frac{\hbar^2}{2I} J(J + 1) \equiv B\hbar(J + 1) \quad \Delta J = \pm 1 \]

The \( J=1 \) state is elevated above the ground by 4.8x10^{-4} eV or, equivalently, 5.5 K \( \rightarrow \) easy to excite in a quiescent cloud.

Within a molecular cloud, excitation of CO to the \( J=1 \) level occurs primarily through collisions with the ambient \( \text{H}_2 \).

The distribution of molecular gas in the Milky Way

molecular gas mass \( \sim \) atomic gas mass:

\[ 2-4 \times 10^9 \, M_\odot \]

(dust/gas mass ratio \( \sim 1\% \))

most of the molecular gas in Giant Molecular Clouds (\( \geq 10^5 \, M_\odot \)),

confined in spiral arms

throughout the disk: small clouds and complexes (\( \sim \) a few \( 10^4 \, M_\odot \))
Physical Properties of Cold Clouds

<table>
<thead>
<tr>
<th>Type</th>
<th>$A_V$</th>
<th>$n_{tot}$</th>
<th>$L$</th>
<th>$T$</th>
<th>$M$</th>
<th>Ex.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffuse</td>
<td>1</td>
<td>500</td>
<td>3</td>
<td>50</td>
<td>50</td>
<td>ζ Oph</td>
</tr>
<tr>
<td>GMC</td>
<td>2</td>
<td>100</td>
<td>50</td>
<td>15</td>
<td>10$^5$</td>
<td>Orion</td>
</tr>
<tr>
<td>Dark:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>complexes</td>
<td>5</td>
<td>500</td>
<td>10</td>
<td>10</td>
<td>10$^4$</td>
<td>Taurus</td>
</tr>
<tr>
<td>individual</td>
<td>10</td>
<td>10$^3$</td>
<td>2</td>
<td>10</td>
<td>30</td>
<td>B1</td>
</tr>
<tr>
<td>Dense cores</td>
<td>10</td>
<td>10$^4$</td>
<td>0.1</td>
<td>10</td>
<td>10</td>
<td>TMC-1  B335</td>
</tr>
</tbody>
</table>

Physical Properties

Both HI and diffuse clouds can persist for long periods by means of pressure balance, because of the presence of a surrounding, more rarefied and warmer medium which prevents the internal (thermal) motion from dispersing the cloud.

In giant molecular clouds, the main cohesive force is typically the cloud’s own gravity, while internal thermal pressure plays only a minor role in the overall force balance.

Within the Milky Way, over 80% of $H_2$ resides in GMCs, which also accounts for most of the star production. Typically, $t$(GMC) $\sim$ 3x10$^7$ yr before it is destroyed by the intense winds from embedded O and B stars. The mass converted into stars is $\sim$ 3% $\Rightarrow$ SFR $\sim$ 2 $M_\odot$ yr$^{-1}$

Every Galactic OB association thus far observed is closely associated with a GMC.
Example of GMC: the Rosette Molecular Cloud

D=1.5 kpc in Monoceros

$^{12}$C$^{16}$O(1-0)

HI envelope

Position-Velocity diagrams

Any map is a two-dimensional projection onto the plane of the sky → structures that are physically distinct are blended together. The radial velocity of the emitting gas can be used as the third coordinate.

The clump velocities within a complex appear to be randomly dispersed about a mean value.
Clump properties

Mean density of complex: \( \sim 60 \text{ cm}^{-3} \)

Typical clump radius: \( \sim 1.5 \text{ pc} \)
Average mass: \( \sim 250 M_{\odot} \)
Average number density: \( \sim 550 \text{ cm}^{-3} \)

Distribution of the clump mass \( M \) above a certain minimum (\( M_{\text{min}} \sim 30 M_{\odot} \)):

\[
N = N_0 \left( \frac{M}{M_{\text{min}}} \right)^{-1.5} \quad M \geq M_{\text{min}}
\]

This relation has also been found in other GMCs and for the masses of the complexes as a whole, suggesting that GMCs may be built up by the agglomeration of many clumps distributed in mass according to the above relation…

Inter-clump medium and atomic constituent

The space between the clumps is occupied by a lower density gas, whose properties are not yet known in detail. The gas is only partially molecular (CO is detected), but there is also presence of HI, seen as absorption dip superposed on the ubiquitous emission from HI. The temperature of this inter-clump medium is \( \sim 20-40 \text{ K} \).

The interclump gas represent a minor mass fraction of the complex. However, complexes have extended and massive atomic envelopes. The size is \( \sim \) several times that of the complex and have comparable mass; the temperature is \( \sim 50-150 \text{ K} \), similar to the HI clouds.
Origin and Demise

The clustering of the complexes along Galactic spiral arms suggests that GMC buildup occurs as gas flows into the potential wells associated with the arms.

The original gas is presumably atomic. Molecular clouds then form inside the condensing medium through their self-shielding from UV radiation.

The observed drop in the H$_2$ surface density between the arms implies that a typical GMC cannot survive as long as the interarm travel time of the Galactic gas (~10$^8$ yr at the solar position).

What destroys the complexes? Possible agents are powerful winds and radiative feedback (ionization and dissociation) associated with massive, embedded stars.

But the observational constraints on GMC lifetimes are very poor at the moment and the relative importance of destruction by feedback versus other processes, such as GMC collisions and accretion are still debated.

In the Rosette Nebula, the O stars have already dispersed much of the adjacent atomic and molecular gas and are driving a thick HI shell into the remainder.

The shell radius is ~18 pc, the expansion velocity is ~5 km/s ➔ the expansion has proceeded for ~4x10$^6$ yr (= age of the partially embedded NGC2244 cluster!!).

Total molecular mass lying within 25 pc of known stellar clusters vs. cluster age. Note the marked decline in the mass by an age of 5x10$^6$ yr and complete disappearance after 5x10$^7$ yr.

The first O stars must appear relatively soon after the formation of a complex, since the majority of GMCs seen today contain a massive association ➔ duration of complex ~ disappearance time.
Vector Calculus

Stahler & Palla use vector calculus in some of their mathematical descriptions. Some of you may not have covered this before. In the homeworks and exams, you will not need to carry out vector calculus, but we summarize here some key features (see also: http://en.wikipedia.org/wiki/Vector_calculus).

Vector operations [edit]

Algebraic operations [edit]
The basic algebraic (non-differential) operations in vector calculus are referred to as vector algebra, being defined for a vector space and then globally applied to a vector field, and consist of:

- **scalar multiplication**
  - multiplication of a scalar field and a vector field, yielding a vector field: \( a \mathbf{V} \);
- **vector addition**
  - addition of two vector fields, yielding a vector field: \( \mathbf{V}_1 + \mathbf{V}_2 \);
- **dot product**
  - multiplication of two vector fields, yielding a scalar field: \( \mathbf{V}_1 \cdot \mathbf{V}_2 \);
- **cross product**
  - multiplication of two vector fields, yielding a vector field: \( \mathbf{V}_1 \times \mathbf{V}_2 \);

There are also two **triple products**:

- **scalar triple product**
  - the dot product of a vector and a cross product of two vectors: \( \mathbf{v}_1 \cdot (\mathbf{v}_2 \times \mathbf{v}_3) \);
- **vector triple product**
  - the cross product of a vector and a cross product of two vectors: \( \mathbf{v}_1 \times (\mathbf{v}_2 \times \mathbf{v}_3) \) or \( (\mathbf{v}_3 \times \mathbf{v}_2) \times \mathbf{v}_1 \);

although these are less often used as basic operations, as they can be expressed in terms of the dot and cross products.

Differential operations [edit]

Vector calculus studies various differential operators defined on scalar or vector fields, which are typically expressed in terms of the del operator (\( \nabla \)), also known as "nabla". The five most important differential operations in vector calculus are:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Notation</th>
<th>Description</th>
<th>Domain/Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gradient</strong></td>
<td>( \text{grad}(f) = \nabla f )</td>
<td>Measures the rate and direction of change in a scalar field.</td>
<td>Maps scalar fields to vector fields.</td>
</tr>
<tr>
<td><strong>Curl</strong></td>
<td>( \text{curl}(\mathbf{F}) = \nabla \times \mathbf{F} )</td>
<td>Measures the tendency to rotate about a point in a vector field.</td>
<td>Maps vector fields to (pseudo)vector fields.</td>
</tr>
<tr>
<td><strong>Divergence</strong></td>
<td>( \text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F} )</td>
<td>Measures the scalar of a source or sink at a given point in a vector field.</td>
<td>Maps vector fields to scalar fields.</td>
</tr>
<tr>
<td><strong>Vector Laplacian</strong></td>
<td>( \nabla^2 \mathbf{F} = \nabla(\nabla \cdot \mathbf{F}) - \nabla \times (\nabla \times \mathbf{F}) )</td>
<td>A composition of the curl and gradient operations.</td>
<td>Maps between vector fields.</td>
</tr>
<tr>
<td><strong>Laplacian</strong></td>
<td>( \Delta f = \nabla^2 f = \nabla \cdot \nabla f )</td>
<td>A composition of the divergence and gradient operations.</td>
<td>Maps between scalar fields.</td>
</tr>
</tbody>
</table>

where the curl and divergence differ because the former uses a cross product and the latter a dot product. \( f \) denotes a scalar field and \( \mathbf{F} \) denotes a vector field. A quantity called the **Jacobian** is useful for studying functions when both the domain and range of the function are multivariable, such as a change of variables during integration.
Virial Theorem Analysis

Fundamental theorem which allow us to assess generally the balance of forces within any structure in hydrostatic equilibrium. In the particular case of clumps within complexes, it will allow us to understand if the magnitude of the observed velocity dispersion is consistent with the internal gravitational field.

Statement of the Theorem

Equation of motion for an inviscid fluid:

\[ \rho \frac{D\mathbf{u}}{Dt} = -\nabla P - \rho \nabla \phi_g + \frac{1}{c} \mathbf{j} \times \mathbf{B} \]

- Full or convective time derivative of the fluid velocity \( \mathbf{u} \)
- Magnetic force per unit volume acting on a current density \( \mathbf{j} \).

Using Ampère’s Law and the vector identity for triple cross products:

\[ \rho \frac{D\mathbf{u}}{Dt} = -\nabla P - \rho \nabla \phi_g + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{1}{8\pi} \nabla |\mathbf{B}|^2 \]

- Tension associated with curved magnetic field lines
- Gradient of a scalar magnetic pressure of magnitude \( |\mathbf{B}|^2/8\pi \)

The existence of tension requires bending of the field lines, while pressure arises from the crowding of the lines, whether they be curved or straight.

The above equation governs the local behaviour of the fluid. To derive a relation between global properties of the gaseous body, let’s form the scalar product of the hydrostatic equation with the position vector \( \mathbf{r} \) and integrate over volume.
Using the mass continuity and Poisson's equations, neglecting the external pressure (valid for the strongly self-gravitating giant complexes, but not for HI or diffuse clouds), repeated integration by parts leads to the virial theorem:

\[
\frac{1}{2} \frac{\partial^2 I}{\partial t^2} = 2T + 2U + W + M
\]

- **Moment of inertia**
- **Total kinetic energy**
- **Energy in random, thermal motion**
- **Gravitational potential energy**
- **Magnetic energy**

\[
I = \int \rho |r|^2 d^3x
\]

\[
T = \frac{1}{2} \int \rho |u|^2 d^3x
\]

\[
U = \frac{3}{2} k_B \int n k_B T d^3x = \frac{3}{2} \int P d^3x
\]

\[
W = \frac{1}{2} \int \rho \phi_g d^3x
\]

\[
M = \frac{1}{8\pi} \int |B|^2 d^3x
\]

The issue here is which of the above terms can balance the gravitational binding energy \(W\). If none, then a typical cloud would be in a state of gravitational collapse.

### Free-fall time

Assuming that a cloud with radius \(R\) and mass \(M\) is in gravitational collapse:

\[
\frac{1}{2} \frac{\partial^2 I}{\partial t^2} \approx - \frac{GM^2}{R}
\]

Approximating \(I\) as \(MR^2\), then \(R\) in the collapsing cloud shrinks by ~ a factor of 2 over a characteristic free-fall time \(t_{ff}\):

\[
t_{ff} \approx \left( \frac{R}{GM} \right)^{1/2} = 7 \times 10^6 \text{ yr} \left( \frac{M}{10^5 M_{\text{sun}}} \right)^{-1/2} \left( \frac{R}{25 \text{ pc}} \right)^{3/2}
\]

Since \(M/R^3 = \rho\), the time scale can also be written as \((G \rho)^{-1/2}\). Conventionally, \(t_{ff}\) is defined by the time for a homogeneous sphere with zero internal pressure to collapse to a point:

\[
t_{ff} \equiv \left( \frac{3\pi}{32G \rho} \right)^{-1/2}
\]
Support of Giant Complexes

If the complexes are in approximate force balance over their lifetimes, we can consider the form of the virial theorem appropriate for longterm stability:

\[ 2T + 2U + W + M = 0 \]

For a cloud is such “virial equilibrium”, what is balancing \( W \)?

1. Thermal energy ??

\[
\frac{U}{|W|} = \frac{MRT}{\mu} \left( \frac{GM^2}{R} \right)^{-1} = 3 \times 10^{-3} \left( \frac{M}{10^5 M_{\odot}} \right)^{-1} \left( \frac{R}{25 \text{ pc}} \right) \left( \frac{T}{15 \text{ K}} \right)
\]

\[ \downarrow \]

NO

2. Magnetic energy ??

\[
\frac{M}{|W|} = \frac{|B|^2 R^3}{6\pi} \left( \frac{GM^2}{R} \right)^{-1} = 0.3 \left( \frac{B}{20 \mu G} \right)^2 \left( \frac{R}{25 \text{ pc}} \right)^4 \left( \frac{M}{10^5 M_{\odot}} \right)^{-2}
\]

Magnetic fields are important!

\[
\rho \frac{Du}{Dt} = -\nabla P - \rho \nabla \phi_g + \frac{1}{c} \mathbf{j} \times \mathbf{B}
\]

The force associated with the magnetic field acts on a fluid element in a direction orthogonal to \( \mathbf{B} \). Thus, any self-gravitating cloud can slide freely along field lines (but no flattening observed!).

The observed scatter in the alignment of the observed \( \mathbf{E} \) vectors indicates the presence of a \textit{random} component of \( \mathbf{B} \), coexisting with the smooth background. The field distortion arises, at least in part, from magnetohydrodynamic (MHD) waves, which may provide the isotropic support preventing the cloud from flattening.
3. Kinetic energy

The bulk velocity within giant clouds stems mostly from the random motions of their clumps ($\Delta V$ is the mean value of the random motion of clumps in GMCs):

$$\frac{T}{|\mathcal{W}|} = \frac{1}{2} M (\Delta V^3)^{-1} G M = 0.5 \left( \frac{\Delta V}{4 \text{ km/s}} \right)^2 \left( \frac{R}{25 \text{ pc}} \right) \left( \frac{M}{10^5 \text{ M}_{\odot}} \right)^{-1}$$

$\Delta V$ is close to the virial velocity, defined as:

$$V_{\text{vir}} = \left( \frac{GM}{R} \right)^{1/2}$$

$\tau_{\text{ff}} = \left( \frac{R^3}{GM} \right)^{1/2}$

$V_{\text{vir}}$ is the velocity of a parcel of gas that traverses the cloud over the $\tau_{\text{ff}}$, i.e. it is the typical speed attained by matter under the influence of the cloud’s internal gravitational field.

$\Delta V$ roughly matches $V_{\text{vir}}$ (or $T \sim |\mathcal{W}|$) over a wide range of sizes.

This approximate equality is consistent with the picture of a swarm of relatively small clumps, each one moving in the gravitational field created by the whole ensemble. The kinetic energy is matched by that of the internal magnetic field, which also has a significant random component.
The line width-size relation (Larson’s law)

\[ \Delta V = \Delta V_0 \left( \frac{L}{L_0} \right)^n \]

\[ n \sim 0.5, \quad \Delta V_0 \sim 1 \text{ km/s for } L_0 = 1 \text{ pc} \]

\[ \Delta V = \Delta V_0 \left( \frac{L}{L_0} \right)^n + \Delta V \approx V_{\text{vir}} \equiv \left( \frac{GM}{R} \right)^{1/2} \]

The column density \( M L^{-2} \) is unchanged from cloud to cloud.

As we consider successively smaller clouds, \( \Delta V \) (the nonthermal component), eventually reaches the ambient thermal speed, whose root-mean-square value is given by \((3RT/\mu)^{1/2}\). This transition is reached at a size scale:

\[ L_{\text{therm}} = \frac{3RTL_0}{\mu \Delta V_0^2} = 0.1 \text{ pc} \left( \frac{T}{10 \text{ K}} \right) \]
An object of size $L_{\text{therm}}$, which also has $U \sim |W|$, is a more quiescent environment than larger clouds with their energetic internal waves. In fact, both clumps within giant complexes and isolated dark clouds do contain distinguishable substructures of this dimension. These entities are the \textit{dense cores} responsible for individual star formation.

With densities exceeding $10^4$ cm$^{-3}$, such structures cannot be probed by the usual $^{12}$C$^{16}$O or even $^{13}$C$^{16}$O lines, both optically thick, but other high density tracers are needed.