ORBITAL ENERGY

$$E = K + U = \frac{1}{2} m v^2 - \frac{GMm}{r}$$

$$\frac{L}{GMm} \vec{v} = \hat{\theta} + \vec{e} \qquad \text{where } \vec{e} = e\hat{j}$$

$$\frac{L^2}{G^2M^2m^2} \vec{v} \cdot \vec{v} = \hat{\theta} \cdot \hat{\theta} + 2e\hat{\theta} \cdot \hat{j} + e^2 \hat{j} \cdot \hat{j}$$

$$\left(\frac{L}{GMm}\right)^2 v^2 = 1 + 2e\cos\theta + e^2$$

$$\text{SO } K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{GMm}{L}\right)^2 (1 + e^2 + 2e\cos\theta)$$

NOTE: K IS MAX AT PERIHELION, $\theta = 0$

$$U = -\frac{GMm}{r} \qquad \text{REMEMBER } r = \frac{L^2}{GMm^2(1 + e\cos\theta)}$$

$$\text{SO } U = -\frac{(GM)^2 m^3}{L^2}(1 + e\cos\theta)$$

$$\text{SO } E = K + U = \left(\frac{GMm}{L}\right)^2 \frac{m}{2}(e^2 - 1)$$

NOTE: IF $e > 1$ $E > 1$ UNBOUND

IF $e = 1$ $E = 0$ MARGINALLY BOUND

IF $e < 1$ $E < 0$ (BOUND)

IF $E = 0$ $K = -U$

$$\tfrac{1}{2} m v_{es}^2 = \frac{GMm}{r} \qquad \text{where } v_{es} = \text{escape velocity}$$

NOW $r = \dfrac{a(1-e^2)}{1+e\cos\theta}$

SO $r(1+e\cos\theta) = a(1-e^2)$

(*) $e\cos\theta = \dfrac{a(1-e^2)-r}{r}$

THIS IS AN EXPRESSION OF θ AS A FUNCTION OF r

NOW

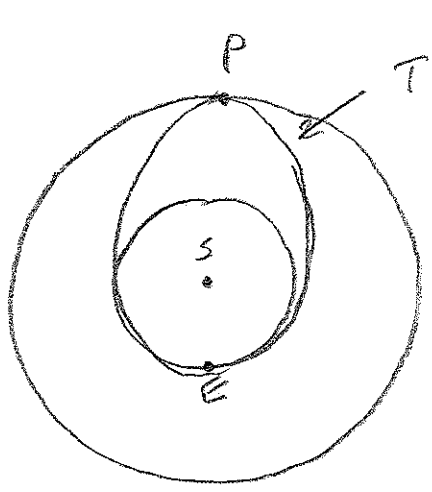
$$v^2 = \frac{2\pi}{m} = \left(\frac{GMm}{L}\right)^2 (1+e^2+2e\cos\theta)$$

Substitute $e\cos\theta$ from (*) above AND $L^2/m^2 = GMa(1-e^2)$ to get

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$ VIS VIVA EQUATION

TRANSFER ORBIT

ASSUME YOU WANT TO SEND A SPACECRAFT TO ANOTHER PLANET.



$a_{to} = \frac{a_E + a_P}{2}$ FIND PERIOD OF TRANSFER ORBIT FROM $P_{to}^2 = a_{to}^3$

TRANSIT TIME IS $\frac{1}{2} P_{to}$

HOW MUCH EXTRA SPEED DO I NEED TO GIVE THE SPACECRAFT?

$v_E = \frac{2\pi a_E}{P_E}$

FOR THE TRANSFER ORBIT CALCULATE THE INSERTION VELOCITY FROM $v_{INS}^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$

THE EXTRA VELOCITY $\Delta v = v_{INS} - v_E$

SIMILARLY YOU NEED TO MATCH THE SATTELITE'S VELOCITY AT APHELION TO MATCH UP WITH THAT OF THE TARGET PLANET.

NOTE: THIS ASSUMES (INCORRECTLY) THAT THE EARTH AND THE TARGET PLANET HAVE NO GRAVITATIONAL AFFECT ON THE SATELLITE.