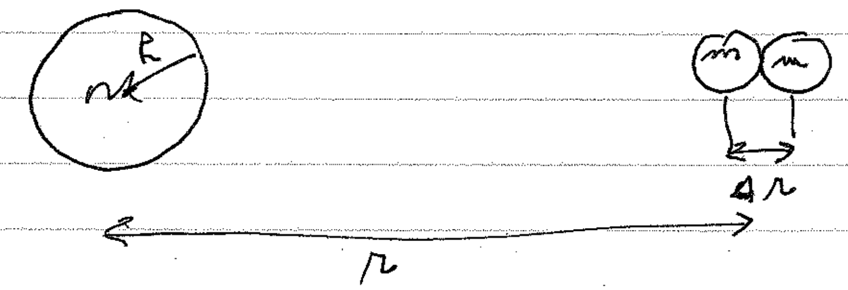
ROCHE'S LIMIT



THE DIFFERENCE IN THE GRAVITATIONAL FORCE OF M ON THE SMALLER MASSES: ΔF IS GIVEN BY

$$\Delta F = \frac{dF}{dr}\Delta r = \frac{2GMm}{r^3}\Delta r$$

THEIR SELF GRAVITY $F = -\frac{Gmm}{(\Delta r)^2}$

WHEN $|\Delta F| = |F|$ $\quad r \equiv r_R$ = ROCHE'S LIMIT

$$\frac{2GMm}{r_R^{\,3}}\Delta r = \frac{Gmm}{(\Delta r)^2} \Rightarrow r_R = \left(\frac{2M}{m}\right)^{1/3}\Delta r$$

EXPRESSING IN TERMS OF ρ (MASS DENSITY)

$$r_R = 2.44\left(\frac{\rho_M}{\rho_m}\right)^{1/3} R$$

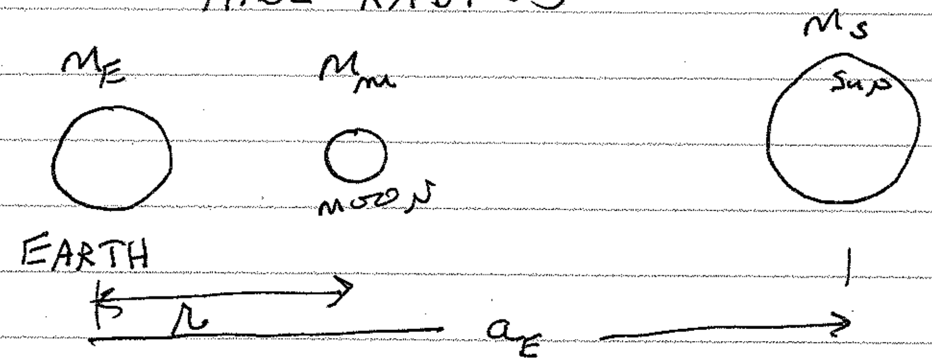
if BOTH bodies have THE SAME DENSITY

$$r_R = 2.44\, R$$

FOR E-MOON SYSTEM $\quad \frac{\rho_E}{\rho_m} = 5/3$

r_R FOR MOON $\simeq 2.9\, R_E$

HILL RADIUS



$$\Delta g = g_m - g_E = \frac{GM_s}{(a_E - r)^2} - \frac{GM_s}{a_E^2}$$

$$(a_E - r)^{-2} = a_E^{-2}\left(1 - \frac{r}{a_E}\right)^{-2} \simeq a_E^{-2}\left(1 + \frac{2r}{a_E}\right)$$

BINOMIAL EXPANSION $\left[\ (x+y)^n = x^n + n x^{n-1} y + \frac{n(n-1)}{2!} x^{n-2} y^2 + \cdots\ \right]$

$$\therefore\ \Delta g \simeq \frac{GM_s}{a_E^2}\left(1 + \frac{2r}{a_E}\right) - \frac{GM_s}{a_E^2} \simeq \frac{2GM_s r}{a_E^3}$$

WHEN $\Delta g = g \quad r = r_H$ (HILL RADIUS)

$$\frac{2GM_s r}{a_E^3} = \frac{GM_E}{r_H^2} \qquad r_H = \left(\frac{M_E}{2M_s}\right)^{1/3} a_E$$

$r_H \simeq 1.7 \times 10^6$ km