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\textbf{ABSTRACT}

The Mid-resolution InfrAReD Astronomical Spectrograph (MIRADAS) is a near-infrared (NIR) multi-object spectrograph for the Gran Telescopio Canarias (GTC). It can simultaneously observe multiple targets selected by 20 identical deployable probe arms with pickoff mirror optics. The bases of the arms are fixed to the multiplexing system (MXS) plate, a circular platform, and arranged in a circular layout with minimum separation between elements of the arms. This document presents the MXS prototype P2a, a full-scale, fully operational prototype of a MIRADAS probe arm. This planar closed-loop mechanism compared to other previous designs offers some advantages specially in terms of stability and from the point of view of optics. Unfortunately, these benefits come at the expense of a more complicated kinematics and an unintuitive arm motion. Furthermore, the cryogenic motor controllers used in prototyping impose severe restrictions in path planing. They negatively impact in the slice of pie approach, a collision-avoidance patrolling strategy that can gives good results in other scenarios. This study is a starting point to define collision-free trajectory algorithms for the 20 probe arms of MIRADAS.

\textbf{Keywords:} Multi-object spectroscopy, probe arms, kinematics, path planning

\section{1. INTRODUCTION}

One of the leading scientific challenges of coming decades is the study of the physical processes that drive galaxy formation and evolution. To answer these open questions, astronomers require the ability to observe several targets at a variety of locations. Multi-Object Spectroscopy (MOS) with multiple deployable Integral Field Units (IFU) allows the simultaneous selection of many science targets and creation of their spectra in a single exposure. KMOS,\textsuperscript{1–3} a NIR integral-field MOS for the very large telescope (VLT) of the European Southern Observatory (ESO), and MIRADAS\textsuperscript{4} are two instruments equipped with such technology.

KMOS is a second-generation instrument in regular use at ESO. It uses 24 robotic pickoff arms with fold mirrors to observe 24 different astronomical locations in a 7.2 arcmin diameter field-of-view (FoV). The 2.8x2.8 arcseconds sky sub-fields selected by the user are anamorphically magnified onto 24 advanced image slicer IFUs. The arms are arranged into two different circular layers, placed above and below of the nominal focal plane. Each layer contains a total of twelve pickoff arms and in both layers the arms are distributed in a circle around the FoV.

The KMOS arm is an open kinematic chain of two degrees of freedom (DoF), each driven by one stepper motor capable of operating in cryogenic. The T-shaped mechanism consist of two tubes and two independent joints that provide to the structure a polar \((r, \varphi)\) motion. A revolute joint in the arm base is responsible of the angular \((\varphi)\) motion. A prismatic (sliding) joint, above the revolute joint and connected to it by a vertical tube, controls the translational or radial \((r)\) motion. The pickoff mirror which diverts the input beam along the arm is located at the end of an horitzontal tube that slides over the prismatic joint. This variable path length of the beam within the arm needs to be compensated by an optical trombone. From the control point of view,

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positioning the tip of the KMOS arm in a given focal plane location is really intuitive due to its polar motion approach. The translation from focal plane cartesian coordinates into theoretical values for the arm control variables is also a straightforward an intuitive process.

MIRADAS is a NIR MOS being developed by a consortium formed by north-american, spanish and mexican institutions. The instrument was selected in 2010 by GTC and the preliminary design proposed was successfully reviewed in November 2012 by an international expert panel. MIRADAS is capable of simultaneously selecting up-to 20 independent and user defined targets in the FoV. This is accomplished using 20 deployable and independently-controlled cryogenic robotic arms with pickoff mirror optics, each feeding a 4.0x1.2 arcsec FoV to the spectrograph. Each probe arm relays light from near the GTC focal plane to its associated macro-slicer IFU. The 20 arms are fixed to the same side of the circular MXS plate and arranged in a circle around the focal plane; see figure 1. As all the probe arms are in a single layer, the MXS plate becomes a very crowded area with minimum separation between mechanical elements. Thus, the trajectories of the 20 arms have to be carefully planned in order to avoid any collision.

The MIRADAS arm is a closed-loop kinematic chain consisting of two rigid tubes, a bar and four joints, being only two of them controlled. The 2-DoF structure produces a planar motion over the plane formed by the MXS plate. This design follows a different approach than KMOS arm. While KMOS arm is based on control motion simplicity, MIRADAS arm is conceived having optics in mind. In MIRADAS, whatever the arm tip position is, the light within the mechanism always follows a fixed optical path length. Furthermore, being a close-loop mechanism gives the arm more stability, an essential characteristic when it is operated upside down.

Calculating optimum collision-free trajectories requires a good knowledge of the MIRADAS arm behaviour based on its geometry and its mechanical constraints. In the following sections, we present the MXS prototype P2a and a geometric model for that 2-DoF mechanism. As we will see, some values of the variables that models the DoFs cannot be used, as they would lead to mechanically unachievable configurations. These pairs must be avoided and determine what we have called the zone of avoidance. The workspace of the arm and its envelope are studied. The theoretical analysis of the arm behavior includes also an analytical and geometric solution for the forward and inverse kinematics problem respectively. Finally, the document presents two patrolling techniques: workspace and slice of pie patrolling.

![Figure 1: The MIRADAS MXS plate contains 20 deployable probe arms. Each of them has a pickoff mirror in its tip used to relay light from a giving point of the instrument FoV to the spectrograph. The MIRADAS FoV is the small circle of radius 125mm. seen in the center of the MXS plate.](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)
In this section we present the MXS prototype P2a and the model of behavior of the arm, paying special attention to its workspace, envelope and kinematics.

2.1 MXS prototype P2a

MXS prototype P2a, that can be seen from an aerial point of view in figure 2a, is a full-scale, fully operational prototype of a MIRADAS probe arm. The mechanism is composed by four joints ($J_1$, $J_2$, $J_3$ and $J_4$), a bar that connects $J_2$ and $J_4$, a short tube that links $J_1$ and $J_3$ and the main tube, with the tip in one end and $J_3$ in the other. The two tubes contain the optics that relay the beam from the pick-off mirror in the tip to a hole below $J_1$. $J_1$ and $J_2$ are revolute joints that are attached to the mechanism base, each of them having its own actuator. The range of motion of $J_1$ and $J_2$ is 180° and 360° respectively. $J_3$ is a rotating joint and $J_4$ has a compound motion: slides over link $Tip - J_3$ and rotates about a shaft perpendicular to this link. Joints $J_3$ and $J_4$ are passive, their position and orientation depend on the orientation of $J_1$ and $J_2$. As can be appreciated in figure 2a, the motion of $J_4$ along the $Tip - J_3$ tube is constrained by two elements: at the left, by a cylinder containing collimating lenses and at the right, by a notch in the tube.

2.2 MXS arm model

The probe arm mechanism is modeled as a close-loop kinematic chain of links connected together by various joints. The arm model, as seen in figure 2b, has four joints ($J_1$, $J_2$, $J_3$ and $J_4$) and four links ($L_1$, $L_2$, $L_3$ and $L_4$) that define a planar motion in the $(x, y)$ cartesian space. The independent joints, $J_1$ and $J_2$, and the dependent, $J_3$, are revolute (R) joints. They are characterized by their angle of rotation ($\theta_1$, $\theta_2$ and $\alpha$ respectively) about z axis, which is perpendicular to the $(x, y)$ plane. $J_1$ and $J_2$ are attached to the arm base, so its position is always the same. $J_4$ is a revolute-prismatic (RP) joint: it is a bearing that rotates about the z axis and also slides along the link $L_3$. The linear motion of $J_4$ has two physical limits represented by $s_1$ and $s_2$. As we can appreciate in figure 2b, the orientation of $J_1$ and $J_2$ univocally determine the position and orientation of all the elements of
the mechanism, including the tip. Thus, the articular variables of \( J_1 \) and \( J_2 \) (\( \theta_1 \) and \( \theta_2 \)) control the two DoF of the probe arm and determine the arm configuration.

The arm has a reference frame located in the center of joint \( J_1 \), representing the local origin of coordinates in the cartesian space for this particular arm. All local computations are made in reference to this frame.

2.3 Configuration Space and Zone of Avoidance

Although the theoretical range of motion of \( J_1 \) and \( J_2 \) is 180° and 360° respectively, only a reduced set of \( (\theta_1, \theta_2) \) pairs satisfy the \( d_{s_1} < d_{34} < d_{s_2} \) constraint. Each of these pairs is a configuration and determines the position and orientation of every mechanism element. The arm configuration space contains all those pairs. The other pairs, those do not satisfy the geometric constrain, belong to the Zone of Avoidance (ZoA). This area, seen in figure 3, must be skipped when planning arm paths.

The value of \( d_{34} \) for a given \( (\theta_1, \theta_2) \) combination can be calculated as follows:

\[
d_{34} = f(\theta_1, \theta_2) = \sqrt{(d_{12} - d_{24} \cos \theta_2 - d_{13} \cos \theta_1)^2 + (d_{24} \sin \theta_2 - d_{13} \sin \theta_1)^2}
\] (1)

Due to the 360° movement of \( J_2 \), the articular space contains infinite replicas of the configuration space and the ZoA distributed along the vertical \( \theta_2 \) axis. Each of them centered in a \( \theta_2 \) that is an integer multiple of 2\( \pi \). As a result, any point of the articular space can be reached going forward or backward along the \( \theta_2 \) axis.

2.4 Workspace and Envelope

The workspace of the mechanism, shown in figure 4a, contains the two-dimensional points of the cartesian space that can be reached by the arm tip. Thus, it determines which points of the FoV can be patrolled by the arm.

We distinguish two regions in the arm workspace, the upper and the lower workspace, seen in figure 4b and in figure 4c. These zones are the result of individually projecting the two sides, the upper and the lower, of the cylinder-shaped volume shown in Figure 4d on the \((x, y)\) plane. That figure represents the relationship

Figure 3: The arm configuration space (white color) is the set of all \( (\theta_1, \theta_2) \) pairs that position \( J_4 \) between \( s_1 \) and \( s_2 \), while the ZoA (grey color) is formed by those pairs that do not satisfy that mechanical constraint. The rotational ranges of joints \( J_1 \) and \( J_2 \) are 180° and 360° respectively. In the articular space, there are as many replicas of the ZoA above and below the one centered at \( \theta_2 = 0 \) as complete clockwise or counter clockwise rotations we apply to \( J_2 \).
(a) The workspace (in dark grey) and envelope (in light grey), both calculated according to the arm frame which is in J1. The locations of J1 and J2 and their range of motion, 180° and 360° respectively, are also represented. (b) The upper workspace appears superposed over the arm workspace. The upper workspace is the result of translating the configurations belonging to the upper side of the cylinder shown in figure 4e into the cartesian space. (c) The lower workspace appears superposed over the arm workspace. The lower workspace is the result of translating the configurations belonging to the lower side of the cylinder shown in figure 4e into the cartesian space.

(d) The 3D workspace. The cylinder-shaped volume shows that some points of the workspace can be reached by two per workspace is the projection of the upper part of the values of d34. As this variable represents the position of cylinder-shaped volume into the (x, y) plane, while the projection of the lower part is known as the lower workspace. by two different (θ1, θ2) pairs.

Figure 4: 2D Workspace, envelope and two different views of the 3D workspace (tip position vs d34).

between the 2D workspace and d34 magnitude. As we can appreciate, an important portion of the workspace presents two different values of d34. This means that, because of the 360° range of motion of J2, some points can be reached by two different arm configurations. One that belongs to the upper workspace and the other to the lower. Finally, we can also see in figure 4 that the most important contribution to the total 2D workspace surface comes from the upper workspace.

The envelope is defined as the curve in the cartesian plane which delimits the area swept out by all the bodies of the arm as the mechanisms executes all configurations. The upper bound of collisions that one arm can experience, as we will discuss in section 3.1, can be obtained by intersecting envelopes.

2.5 Kinematics

The translation of one point in the arm configuration space into one point in the cartesian space and viceversa is achieved by what is known as forward kinematics and inverse kinematics. In the following sections we present
an analytical solution for forward kinematics, while a geometric solution is found for inverse kinematics. The study does not consider joint velocity or acceleration.

2.5.1 Forward kinematics

Given a point of the configuration space, the forward kinematics calculates the cartesian space position of the tip with respect to the arm frame. According to the model presented in figure 2b, the \((x, y)\) tip position can be obtained from the following equations:

\[
x_{\text{tip}} = f(\theta_1, \theta_2) = d_{13} \cos \theta_1 + d_{3\text{tip}} \cos \alpha \tag{2a}
\]
\[
y_{\text{tip}} = f(\theta_1, \theta_2) = d_{13} \sin \theta_1 + d_{3\text{tip}} \sin \alpha \tag{2b}
\]

where \(\alpha\) is a function of \(\theta_1\) and \(\theta_2\) that can be calculated from the loop-closure expression:

\[
L_1 + L_{34} + L_2 + L_4 = 0 \tag{3}
\]

Expanding the vectorial equation (3) into its \(x\) and \(y\) components:

\[
d_{13} \cos \theta_1 + d_{34} \cos \alpha + d_{24} \cos \theta_2 - d_{12} = 0 \tag{4a}
\]
\[
d_{13} \sin \theta_1 + d_{34} \sin \alpha - d_{24} \sin \theta_2 = 0 \tag{4b}
\]

and combining equation (4a) and equation (4b), we find that:

\[
\alpha = f(\theta_1, \theta_2) = \arctan \left( \frac{-d_{13} \sin \theta_1 + d_{24} \sin \theta_2}{d_{12} - d_{13} \cos \theta_1 - d_{24} \cos \theta_2} \right) \tag{5}
\]

Finally, using trigometric equivalent expressions, the \((x, y)\) tip position can be expressed as:

\[
x_{\text{tip}} = f(\theta_1, \theta_2) = d_{13} \cos \theta_1 + d_{3\text{tip}} \frac{1}{\sqrt{1 + u^2}} \tag{6a}
\]
\[
y_{\text{tip}} = f(\theta_1, \theta_2) = d_{13} \sin \theta_1 + d_{3\text{tip}} \frac{u}{\sqrt{1 + u^2}} \tag{6b}
\]

where \(u\) is:

\[
u = f(\theta_1, \theta_2) = \frac{-d_{13} \sin \theta_1 + d_{24} \sin \theta_2}{d_{12} - d_{13} \cos \theta_1 - d_{24} \cos \theta_2} \tag{7}
\]

2.5.2 Inverse kinematics

The inverse kinematics finds the values of \(\theta_1\) and \(\theta_2\) that position the tip in a given \((x, y)\) coordinate. Because the forward equations are nonlinear functions of the joint variables, obtaining an analytical solution is a difficult task. Here, we present a geometric solution based on intersections between circles and lines.

According to the model in figure 2b, any arm configuration has to satisfy two conditions: the distance between \(J_1-J_3\) has to be \(d_{13}\) and the distance \(J_3-\text{tip}\) has to be \(d_{3\text{tip}}\). Thus, the feasible positions of \(J_3\) can be obtained by intersecting two circles, one of radius \(d_{13}\) with center at \(J_1\) and other of radius \(d_{3\text{tip}}\) with center at the tip. Due to the limited range of motion of \(J_1\), only those solutions of \(J_3\) that produce a \(\theta_1\) in the interval of \([0^\circ, 180^\circ]\) will be valid.

Once determined a valid position for \(J_3\), we can draw a line from that point to the tip. This straight line represents the link \(L_3\). As \(J_4\) must be in \(L_3\), its positions can be obtained by intersecting the line that models \(L_3\) and a circle of radius \(d_{24}\) with center \(J_2\). If any of the two solutions is not in the segment limited by the stop points \(s_1\) and \(s_2\), it has to be rejected.
2.5.3 Motor controller constraints and desired trajectories

Prototype P2a uses Phytron cryogenic stepper motors to drive $J_1$ and $J_2$ joints. These commercial motors were selected because in previous experiences they demonstrated their reliability. Unfortunately, their Phytron MCC$^{2,6}$ ethernet controller presents a particular behavior: it does not buffer incoming commands. If the device receives a new command before the current is complete, it suddenly aborts its execution and proceeds with the justly arrived command.

The geometric path that a mechanism follows, either in cartesian or configuration space, usually is expressed in a parametric form and it can be linear, circular or a section of analytical functions. A common practice is discretizing the path into several waypoints or knots based on a uniform time interval. These knots are then connected together through straight-lines. The MCC2 controller only provide commands for generating linear paths between two articular space points. Each command of this type always includes three stages: an acceleration from the current position, a constant speed motion and, finally, a deceleration before reaching its destination. Therefore, the more intermediate points in a trajectory, the more start-stop sequences will experience the joints.

According to the previously described constraints, we avoid aborting any motion command as the tip can be left at an undetermined position. Furthermore, to preserve the motors lifetime, we only consider trajectories that minimize the number of waypoints between the origin and the destination. The ideal case is determining linear trajectories in the configuration space.

2.5.4 Desired trajectories

The parametric equation of a straight-line trajectory that connects two points in the joint space is:

$$\begin{align*}
\theta_1(t) &= v_{\theta_1} t + \theta_{1i} \\
\theta_2(t) &= v_{\theta_2} t + \theta_{2i}
\end{align*}$$

(8)

where $\theta_1(t)$ and $\theta_2(t)$ specify the trajectory of each joint. The parameters $\theta_{1i}$ and $\theta_{2i}$ are the initial joint positions of the arm and, $v_{\theta_1}$ and $v_{\theta_2}$ are the number of motor steps per second.

If we want that both joints reach at the same moment ($t$) the final position $(\theta_{1f}, \theta_{2f})$, the theoretical speeds of each of them can be calculated as follows:

$$\begin{align*}
v_{\theta_1} &= \frac{\theta_{1f} - \theta_{1i}}{t} \quad \text{(9a)} \\
v_{\theta_2} &= \frac{\theta_{2f} - \theta_{2i}}{t} \quad \text{(9b)}
\end{align*}$$

Equation (8) assumes that $v_{\theta_1}$ and $v_{\theta_2}$ are continuous magnitudes, while the controller only accepts a discrete set of values. Thus, the theoretical values obtained in equation (9a) and equation (9b) have to be approximated to the values specified in the Phytron MCC2 manual.

3. THE ARMS IN THE MXS PLATE

MIRADAS has a total of 20 probe arms distributed around a circular platform, the MXS plate. Each of these mechanisms observes a different object in the instrument FoV. As a result of the high density of arms, as seen in figure 1 on page 2, and the mechanism geometry, there is a high risk of collisions between mechanical elements. The number of potential collisions depends, among other factors, on the location of the 20 targets, the area of the FoV that patrols each arm and the target-to-arm assignment. Here we discuss two patrolling strategies: workspace patrolling and Slice of Pie (SoP) patrolling.
(a) The full-scale MIRADAS MXS plate is divided into 5 equal portions, delimited by the grey dotted lines. Each of them contains a probe arm, represented here by its envelope and workspace. As can be appreciated, although MIRADAS has a total of 20 arms, this 5 arm distribution can patrol the instrument FoV (black dashed circle).

Figure 5: Aerial views of MIRADAS MXS plate with two different arm distributions.

(b) The MXS plate is divided into 20 equal portions containing each one probe arm. There are three envelopes: the arm in blue and the envelopes of its neighbors, arm in green and arm in red. The size of the area resulting from intersecting the three envelopes, gives an idea of the collisions that can happen with only these arms.

Figure 6: Each colored area of the arm envelope specifies the maximum number of collisions it can experiment when passing through that zone. The grey dashed circle centered at (0, 0) represents the FoV instrument.
3.1 Workspace patrolling

In workspace patrolling each arm patrols the points of its workspace that are in the instrument FoV. With this approach, the whole focal plane can be serviced with only 5 arms; see figure 5a. Furthermore, with the layout shown some areas are patrolled by more than one arm. If we now consider the MXS plate containing a total of 20 arms distributed in its periphery, the number of workspace superpositions will increase considerably. Just as an example, in figure 5b, we can see the envelope of one arm and those belonging to the adjacent neighbors. This patrolling strategy gives rise to several target-to-arm assignments, which increases the number of trajectories to consider and also the number of collisions to check.

Two arms can collide with each other if their envelopes overlap. Thus, the upper bound of collisions of one arm can be determined by successively intersecting its envelope with the other arms envelopes. After exploring the whole tree of possible intersections, we find that one arm can suffer up to 524287 intersections.

From the point of view of arms control, it is useful to know which are the areas of one arm envelope with higher collision probability. The path planning algorithm can intensively use this information to avoid generating paths that pass through them. Figure 6 shows one arm envelope divided into different colored area, each of them represents the number of collisions that can happen there. As we can appreciate, the number of collisions in the FoV goes from 7 to 19. This figure increases as the arm gets closer to the FoV center, a point that can be reached by the 20 arms.

3.2 SoP patrolling

In this strategy the field-of-regard of MIRADAS is divided into 20 identical areas. Each area or SoP is a subset of the arm workspace. A given SoP is patrolled by only one arm and each arm patrols always the same SoP. With this approach, SoP patrolling reduces the search tree that arises from workspace patrolling.

In general, SoP patrolling is a technique that avoids collisions and makes control simpler. As long as the following conditions are satisfied, the concept of SoP can ensure there are no collisions between arms:

1. No over-lapping between the different SoPs.
2. The surface resulting from the sum of all SoPs covers the whole FoV.
3. Feasible trajectories can be computed between any two points of one arm SoP.
4. There is no intersections between all the SoP envelopes.

where SoP envelope is the area swept out by the mechanism when visiting all the points in the SoP.

The previous conditions can be accomplished if each arm works as a revolute-prismatic (RP) open kinematic chain. This mechanism composed of two joints describes a polar \((r, \phi)\) motion in the cartesian plane. Due to the geometry of the MIRADAS arm, the high density of mechanical elements in the MXS plate and the trajectories restrictions imposed by the motor controller, satisfying those conditions becomes more difficult.

From the point of view of the field-of-regard, the logical choice is a triangular shaped SoP equivalent to a 18° region. Unfortunately, this triangle translates into a configuration space area with a curved boundary. Thus, two points of the configuration space SoP can not be connected with a single straight line, the desired trajectory as discussed in section 2.5.4.

Considering the constraints imposed by the motor controller, a good choice is that SoP that translated into the articular space becomes a polygon. Thus, a reasonable approach is focusing first on acceptable figures in the articular space and then analyzing its shape in the cartesian plane. A polygon in the configuration space results in a cartesian curved shape; see equations in section 2.5.1. This shape can be eligible as SoP if satisfies, at least, the second point of the previous list: the surface resulting from the sum of all SoPs covers the whole FoV.

When forming polygons in the configuration space, the vertices have to be carefully chosen. If not, the resulting cartesian shape can cross itself. To illustrate this, we construct a triangle whose vertices represent singular points in cartesian space: the FoV center and two points that form a 18° arc in the FoV boundary. If we translate these cartesian points into articular points and connect them by straigh-lines, we obtain the triangle.
(a) A SoP that crosses itself. Each mark in the SoP boundary represents an increment of 10°. The SoP is formed by three curves that connect three points: the FoV center (point (0, 0)), a point in the FoV boundary at 10° and another at -10°. The intersection between the curves is due to one point of the SoP boundary belongs to the upper workspace, while the others to the lower, as can be seen in figure 7b.

(b) The area in the center of the figure delimited by the grey dashed lines contains the configurations belonging to the lower workspace, the other points belong to the upper workspace. The triangles represent the configurations that position the arm tip in the corresponding degree marks on the FoV boundary shown in figure 7a. There are some FoV boundary degrees that can be reached by two (θ₁, θ₂) pairs, one in the upper workspace and other in the lower. Those degrees that can not be reached are in the ZOA or are not represented in the figure. The SoP here is a perfect triangle, with the center of the FoV in the upper workspace and the 10° and -10° points in the lower.

Figure 7: A SoP boundary containing points of the different regions of the workspace.
(a) One typical MIRADAS SoP for arm$_0$ that does not cross itself and produces a polygon in the articular space. Due to the particular geometry of the arm, it needs four vertices to produce a closed shape. As the blue curve does not end in the FoV boundary (it is close to the $10^\circ$ mark), there is an area close to that boundary that is unpatrolled.

(b) Representation of the MIRADAS FoV surface and the SoPs belonging to three adjacent arms: arm$_0$, arm$_1$ and arm$_{19}$. Although part of the FoV area that can not patrol arm$_0$ is now serviced by its adjacent arms, there are still some FoV points that can not be observed.

(c) The translation of the SoP in figure 8a into the configuration space. All the vertices of the quadrilateral belongs to the same region of the workspace, the upper. Here we can see why the blue curve in figure 8a can not end at the $10^\circ$ mark of the FoV. One of the two $10^\circ$ configurations is in the upper right part of the ZoA and the other in the lower workspace.

Figure 8: A typical SoP boundary, where some FoV areas are not patrolled.
in figure 7b. In figure 7a we can see the triangle converted to the cartesian plane. The intersection between the two curves is the result of connecting two points of the configuration space that belong to different regions of the arm workspace. One of them belongs to the upper workspace and the other to the lower. To avoid undesirable intersections between cartesian space curves, all the vertices of the polygon have to belong to the same region of the workspace.

In figure 8a we can see a typical SoP that produces a quadrilateral in the configuration space. It is a good example of the limitations of this patrolling approach when defining SOPs that take into account the constrains of the motor controller. In the particular case shown in figure 8c, the -10° point of the FoV boundary has only one feasible configuration, which belongs to the lower workspace. Thus, it can not be used along with the other articual vertex because these are in the upper workspace. As a result, one of the SoP curves does not ends in the FoV boundary, leaving some points of the FoV unpatrolled. When combining the SoP of arm0 with the SoPs of arm1 and arm19, some of the points that could not be patrolled by arm0 are now serviced. Unfortunately, that is not enough. Still, several areas in the FoV perifery can not be observed, as seen in figure 8b, because they are not patrolled by any of the 20 arms.

4. CONCLUSION

Prior to the design of a MIRADAS trajectories planning algorithm, a detailed probe arm study is required. This paper has carried out part of that initial analysis. Specifically, we have presented a kinematic model, the envelope and the workspace of the MIRADAS probe arm. We have seen that due to the 360° range of motion of one mechanism active joint, the same point in the workspace can usually be reached by two different configurations. One belonging to what we have called upper workspace and the other to lower workspace. In addition, we have also shown that the upper workspace contains almost all the points of workspace.

The document has introduced two patrolling strategies. The workspace patrolling is the approach that enables future planning algorithm to obtain better solutions. That is because it works with a very large search tree. On the other hand, the path planning can require the use of heuristics for fast convergence to an optimal solution. As a first approximation to reducing the search tree, the algorithm can consider only configurations belonging to the upper workspace. Implementing the SoP patrolling strategy with other mechanisms, such as polar motion manipulators, is an intuitive process that can gives good results, but in MIRADAS it becomes a challenging task. The options for a proper SOP are drastically reduced, since the particular arm geometry and the restrictions in trajectories imposed by the motor controllers of the prototype. Although this strategy is still usable with the presented prototype, it does not ensure that, as shown, all points of the FoV can be observed. Experimenting with alternative motor controllers in order to obtain more convenient SoPs remains for future work. We are working in determining the SoP envelope, which will help us in comparing the two presented patrolling approaches in terms of possible collisions.

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